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Parallelizable Calculation of Observables Values on Analog Quantum Computer

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Abstract

The superiority of hypothetical quantum computers is not due to faster calculations but due to different schemes of calculations running on special hardware. The core of quantum computing follows the way a state of a quantum system is defined when basic things interact with each other. In conventional approach it is implemented through tensor product of qubits. In the geometric algebra formalism simultaneous availability of all the results for non-measured observables is based on the definition of states as points on three-dimensional sphere.

Keywords

Geometric Algebra, Wave Functions, Entanglement, Maxwell Equations, Three-Dimensional Sphere, States, Observables, Measurements, GPU, Multithreading, OpenCL

1. Introduction: What Is Entanglement in Conventional Approach and in the Geometric Algebra Approach?

In conventional quantum mechanics entanglement means that multiple objects share a single quantum state. Entanglement is the amount by which multiple objects share a quantum state. They remain indeterminate until they are disentangled by a measurement.

The simplest quantum mechanical state, qubit, reads:

$$C^2 \ni \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = z_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + z_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = z_1 |0\rangle + z_2 |1\rangle$$

$$z_1 = x_1 + iy_1, \quad z_2 = x_2 + iy_2, \quad \|x_1 + iy_1\|^2 + \|x_2 + iy_2\|^2 = 1$$

It has just two observable "things" after measurement, say "up" for $|0\rangle$ and "down" for $|1\rangle$, with probabilities z_1^2 and z_2^2. The qubit dimension two is the number of different observable things available after making a measurement on the particle.

In the case of two particles the vector space C^2 is generalized to density matrix defined on tensor product $C^2 \otimes C^2$. The appropriateness of tensor products is that the tensor product itself captures all the ways that basic things can "interact" with each other [1]. Quantum entanglement means that aspects of one particle of an entangled pair depend on aspects of the other particle. Only when the measurement occurs does the quantum state "collapse" make instantaneously collapsing the other particle state.

That's crucial words: "instantaneously collapsing". Common wisdom considers that as physically real transformation of the other particle unknown state into the known one.

The scheme suggested in the geometric algebra approach is based on manipulation and transferring of quantum states as operators acting on observables. Wave functions act in that context on static G_3^+ elements through measurements, creating "particles", see [2].

When assuming that observables are identified by points on $\mathbb{S}^3$ we get that measurements of any number of observables by arbitrary set of wave functions are simultaneously available, see [1] [3] [4].

This is a geometrically clear and unambiguous explanation of strict connectivity of the results of measurements instead of quite absurd "entanglement" in conventional quantum mechanics.

2. Maxwell Equation in Geometric Algebra

Without charges and currents the Maxwell equation is:

$$\left(\partial_t + \nabla\right) F = 0 \tag{2.1}$$

The circular polarized electromagnetic waves are the only type of waves following from the solution of Maxwell equations in free space done in geometric algebra terms. Indeed, let's take the electromagnetic field in the form:

$$F = F_0 \exp\left[I_S \left(\omega t - k \cdot r \right) \right] \tag{2.2}$$

requiring that it satisfies (2.1).

Element F_0 in (2.2) is a constant element of geometric algebra G_3 and I_S is unit value bivector of a plane S in three dimensions, generalization of the imaginary unit [3]. The exponent in (2.2) is unit value element of G_3^+:

$$\mathrm{e}^{I_S \varphi} = \cos\varphi + I_S \sin\varphi, \quad \varphi = \omega t - k \cdot r$$

Solution of (2.1) should be sum of a vector (electric field e) and bivector (magnetic field $I_3 h$):

$$F = e + I_3 h$$

For a plane S in three dimensions Maxwell Equation (2.1) has two solutions [5]:

- $F_+ = \left(e_0 + I_3 h_0\right) \exp\left[I_S \left(\omega t - k_+ \cdot r \right) \right]$, with $\hat{k}_+ = I_3 I_S$, $\hat{e}\hat{h}\hat{k}_+ = I_3$, and the triple

$\{\hat{e},\hat{h},\hat{k}_+\}$ is right hand screw oriented, that's rotation of $\hat{e}$ to $\hat{h}$ by π/2 gives movement of *right hand screw* in the direction of $k_+ = |k| I_3 I_S$.

- $F_- = (e_0 + I_3 h_0)\exp\left[I_S(\omega t - k_- \cdot r)\right]$, with $\hat{k}_- = -I_3 I_S$, $\hat{e}\hat{h}\hat{k}_- = -I_3$, and the triple $\{\hat{e},\hat{h},\hat{k}_-\}$ is left hand screw oriented, that's rotation of $\hat{e}$ to $\hat{h}$ by $\{\hat{e},\hat{h},\hat{k}_-\}$ gives movement of *left hand screw* in the direction of $k_- = -|k| I_3 I_S$ or, equivalently, movement of *right hand screw* in the opposite direction, $-k_-$.
- e_0 and h_0, initial values of *e* and *h*, are arbitrary mutually orthogonal vectors of equal length, lying on the plane *S*. Vectors $k_\pm = \pm|k_\pm| I_3 I_S$ are normal to that plane. The length of the "wave vectors" $|k_\pm|$ is equal to angular frequency ω.

Maxwell Equation (2.1) is a linear one. Then any linear combination of F_+ and F_- saving the structure of (2.2) will also be a solution.

Let's write:

$$\begin{cases} F_+ = (e_0 + I_3 h_0)\exp\left[I_S\omega\left(t - (I_3 I_S)\cdot r\right)\right] = (e_0 + I_3 h_0)\exp\left[I_S\omega t\right]\exp\left[-I_S\left[(I_3 I_S)\cdot r\right]\right] \\ F_- = (e_0 + I_3 h_0)\exp\left[I_S\omega\left(t + (I_3 I_S)\cdot r\right)\right] = (e_0 + I_3 h_0)\exp\left[I_S\omega t\right]\exp\left[I_S\left[(I_3 I_S)\cdot r\right]\right] \end{cases} \tag{2.3}$$

Then for arbitrary (real[1]) scalars λ and μ:

$$\lambda F_+ + \mu F_- = (e_0 + I_3 h_0)\mathrm{e}^{I_S\omega t}\left(\lambda \mathrm{e}^{-I_S\left[(I_3 I_S)\cdot r\right]} + \mu \mathrm{e}^{I_S\left[(I_3 I_S)\cdot r\right]}\right) \tag{2.4}$$

is solution of (2.1). The item in the second parenthesis is weighted linear combination of two states with the same phase in the same plane but opposite sense of orientation. The states are strictly coupled, entangled if you prefer, because bivector plane should be the same for both, does not matter what happens with that plane.

Arbitrary linear combination (2.4) can be rewritten as:

$$\lambda \mathrm{e}^{I^+_{Plane}\varphi^+} + \mu \mathrm{e}^{I^-_{Plane}\varphi^-} \tag{2.5}$$

where

$$\varphi^\pm = \cos^{-1}\left(\frac{1}{\sqrt{2}}\cos\omega\left(t \mp \left[(I_3 I_S)\cdot r\right]\right)\right),$$

$$\begin{aligned} I^\pm_{Plane} = {} & I_S \frac{\sin\omega\left(t \mp \left[(I_3 I_S)\cdot r\right]\right)}{\sqrt{1+\sin^2\omega\left(t \mp \left[(I_3 I_S)\cdot r\right]\right)}} + I_{B_0}\frac{\cos\omega\left(t \mp \left[(I_3 I_S)\cdot r\right]\right)}{\sqrt{1+\sin^2\omega\left(t \mp \left[(I_3 I_S)\cdot r\right]\right)}} \\ & + I_{E_0}\frac{\sin\omega\left(t \mp \left[(I_3 I_S)\cdot r\right]\right)}{\sqrt{1+\sin^2\omega\left(t \mp \left[(I_3 I_S)\cdot r\right]\right)}} \end{aligned}$$

The triple of unit value basis orthonormal bivectors $\{I_S, I_{B_0}, I_{E_0}\}$ is comprised of the I_S bivector, dual to the propagation direction vector; I_{B_0} is dual to initial vector of magnetic field; I_{E_0} is dual to initial vector of electric field. The expression (2.5) is linear combination of two geometric algebra states, *g*-qubits.

[1]Remember, in the current theory scalars are real ones. "Complex" scalars have no sense.

Linear combination of the two equally weighted basic solutions of the Maxwell equation F_+ and F_-, $\lambda F_+ + \mu F_-$ with $\lambda = \mu = 1$ reads:

$$\lambda F_+ + \mu F_- \big|_{\lambda=\mu=1} = 2\cos\omega\left[\left(I_3 I_S\right)\cdot r\right]\left(\frac{1}{\sqrt{2}}\cos\omega t + I_S\frac{1}{\sqrt{2}}\sin\omega t + I_{B_0}\frac{1}{\sqrt{2}}\cos\omega t + I_{E_0}\frac{1}{\sqrt{2}}\sin\omega t\right) \tag{2.6}$$

where $\cos\varphi = \frac{1}{\sqrt{2}}\cos\omega t$ and $\sin\varphi = \frac{1}{\sqrt{2}}\sqrt{1+\left(\sin\omega t\right)^2}$. It can be written in standard exponential form $\cos\varphi + \sin\varphi I_B = e^{I_B\varphi}$.[2]

I will call such *g*-qubits ***spreons*** because they spread over the whole three-dimensional space for all values of time and instantly change under Clifford translations over the whole three-dimensional space for all values of time, along with the results of measurement of any observable.[3]

3. Measurement of Observables by Spreons

Measurement of any observable $C_0 + C_1 B_1 + C_2 B_2 + C_3 B_3$ (actually Hopf fibration) by a state $\alpha + \beta_1 B_1 + \beta_2 B_2 + \beta_3 B_3$ in the current formalism is, see, for example, [6]:

$$\begin{aligned} &C_0 + C_1 B_1 + C_2 B_2 + C_3 B_3 \xrightarrow{\alpha+\beta_1 B_1+\beta_2 B_2+\beta_3 B_3} \\ &C_0 + \left(C_1\left[\left(\alpha^2+\beta_1^2\right)-\left(\beta_2^2+\beta_3^2\right)\right] + 2C_2\left(\beta_1\beta_2 - \alpha\beta_3\right) + 2C_3\left(\alpha\beta_2 + \beta_1\beta_3\right)\right)B_1 \\ &+\left(2C_1\left(\alpha\beta_3 + \beta_1\beta_2\right) + C_2\left[\left(\alpha^2+\beta_2^2\right)-\left(\beta_1^2+\beta_3^2\right)\right] + 2C_3\left(\beta_2\beta_3 - \alpha\beta_1\right)\right)B_2 \\ &+\left(2C_1\left(\beta_1\beta_3 - \alpha\beta_2\right) + 2C_2\left(\alpha\beta_1 + \beta_2\beta_3\right) + C_3\left[\left(\alpha^2+\beta_3^2\right)-\left(\beta_1^2+\beta_2^2\right)\right]\right)B_3 \end{aligned}$$

In the case of spreon (2.6):

$$B_1 = I_S,\quad B_2 = I_{B_0},\quad B_3 = I_{E_0},$$

$$\alpha = 2\cos\omega\left[\left(I_3 I_S\right)\cdot r\right]\frac{1}{\sqrt{2}}\cos\omega t$$

$$\beta_1 = 2\cos\omega\left[\left(I_3 I_S\right)\cdot r\right]\frac{1}{\sqrt{2}}\sin\omega t$$

$$\beta_2 = 2\cos\omega\left[\left(I_3 I_S\right)\cdot r\right]\frac{1}{\sqrt{2}}\cos\omega t$$

$$\beta_3 = 2\cos\omega\left[\left(I_3 I_S\right)\cdot r\right]\frac{1}{\sqrt{2}}\sin\omega t,$$

and we get a G_3^+ element spreading through the three-dimensional space for all values of the time parameter *t*. It is G_3^+ element spreading through the

[2]Good to remember that the two basic solutions F_+ and F_- differ only by the sign of $I_3 I_S$, which is caused by orientation of I_S that in its turn defines if the triple $\left\{\hat{E}, \hat{H}, \pm I_3 I_S\right\}$ is right-hand screw or left-hand screw oriented.

[3]The two received solutions are similar in their form to the Majorana operators, though with a bivector instead of formal imaginary unit. Thus, not surprising that the following computational scheme represents more general software simulation which gets around the problem of physical implementation of Majorana fermions.

three-dimensional space for all values of time parameter t:

$$4\cos^2\omega\left[\left(I_3 I_S\right)\cdot r\right]\left[C_0 + C_3 I_S + \left(C_1\sin 2\omega t + C_2\cos 2\omega t\right)I_{B_0} + \left(C_2\sin 2\omega t - C_1\cos 2\omega t\right)I_{E_0}\right] \quad (3.1)$$

Geometrically, that means that the measured observable is rotated by $\frac{\pi}{2}$ in the I_{B_0} plane, such that the $C_3 I_S$ component becomes orthogonal to plane I_S and remains unchanged. Two other components became orthogonal to I_{B_0} and I_{E_0} and continue rotating in I_S with angular velocity $2\omega t$. The factor $4\cos^2\omega\left[\left(I_3 I_S\right)\cdot r\right]$ defines the dependency of that transformed values through all points of the three-dimensional space.

In the current scheme any test observable can be placed anywhere into continuum of the (t,r) dependent values of the spreon state. The above formula (3.1) gives the result of measurements simultaneously at all points (t,r).

Assume we have two observables, $C_0^1 + C_1^1 B_1 + C_2^1 B_2 + C_3^1 B_3$ and $C_0^2 + C_1^2 B_1 + C_2^2 B_2 + C_3^2 B_3$. Write them in exponential form:

$$\begin{aligned} C^1 &\equiv C_0^1 + C_1^1 B_1 + C_2^1 B_2 + C_3^1 B_3 \\ &= \left|C^1\right|\left(\frac{C_0^1}{\left|C^1\right|} + \frac{\left|C_B^1\right|}{\left|C^1\right|}\frac{C_1^1 B_1}{\left|C_B^1\right|} + \frac{\left|C_B^1\right|}{\left|C^1\right|}\frac{C_2^1 B_2}{\left|C_B^1\right|} + \frac{\left|C_B^1\right|}{\left|C^1\right|}\frac{C_3^1 B_3}{\left|C_B^1\right|}\right) \\ &= \left|C^1\right|\left(\cos\varphi_1 + \sin\varphi_1 S_1\right) = \left|C^1\right| e^{S_1\varphi_1} \end{aligned}$$

where $\left|C^1\right| = \sqrt{\left(C_0^1\right)^2 + \left(C_1^1\right)^2 + \left(C_2^1\right)^2 + \left(C_3^1\right)^2}$, $\left|C_B^1\right| = \sqrt{\left(C_1^1\right)^2 + \left(C_2^1\right)^2 + \left(C_3^1\right)^2}$,

$$\cos\varphi_1 = \frac{C_0^1}{\left|C^1\right|},\quad \sin\varphi_1 = \frac{\left|C_B^1\right|}{\left|C^1\right|},\quad S_1 = \frac{C_1^1 B_1}{\left|C_B^1\right|} + \frac{C_2^1 B_2}{\left|C_B^1\right|} + \frac{C_3^1 B_3}{\left|C_B^1\right|}$$

$$\begin{aligned} C^2 &\equiv C_0^2 + C_1^2 B_1 + C_2^2 B_2 + C_3^2 B_3 \\ &= \left|C^2\right|\left(\frac{C_0^2}{\left|C^2\right|} + \frac{\left|C_B^2\right|}{\left|C^2\right|}\frac{C_1^2 B_1}{\left|C_B^2\right|} + \frac{\left|C_B^2\right|}{\left|C^2\right|}\frac{C_2^2 B_2}{\left|C_B^2\right|} + \frac{\left|C_B^2\right|}{\left|C^2\right|}\frac{C_3^2 B_3}{\left|C_B^2\right|}\right) \\ &= \left|C^2\right|\left(\cos\varphi_2 + \sin\varphi_2 S_2\right) = \left|C^2\right| e^{S_2\varphi_2} \end{aligned}$$

where $\left|C^2\right| = \sqrt{\left(C_0^2\right)^2 + \left(C_1^2\right)^2 + \left(C_2^2\right)^2 + \left(C_3^2\right)^2}$, $\left|C_B^2\right| = \sqrt{\left(C_1^2\right)^2 + \left(C_2^2\right)^2 + \left(C_3^2\right)^2}$, $\cos\varphi_2 = \frac{C_0^2}{\left|C^2\right|}$, $\sin\varphi_2 = \frac{\left|C_B^2\right|}{\left|C^2\right|}$, $S_2 = \frac{C_1^2 B_1}{\left|C_B^2\right|} + \frac{C_2^2 B_2}{\left|C_B^2\right|} + \frac{C_3^2 B_3}{\left|C_B^2\right|}$.

The observable C^2 can be written as:

$$C^2 = \frac{\left|C^2\right|}{\left|C^1\right|} C^1 e^{-S_1\varphi_1} e^{S_2\varphi_2},$$

and then its measurement by any g-qubit $e^{S\varphi}$ reads:

$$e^{-S\varphi} C^2 e^{S\varphi} = e^{-S\varphi}\frac{\left|C^2\right|}{\left|C^1\right|} C^1 e^{S\varphi} e^{-S\varphi} e^{-S_1\varphi_1} e^{S_2\varphi_2} e^{S\varphi} \quad (3.2)$$

that is, up to the factor $\frac{|C^2|}{|C^1|}$, the result of measurement of C^1, multiplied by the result of measurement of $e^{-S_1\varphi_1}e^{S_2\varphi_2}$.

4. Simultaneous Measurement of Observables by Spreons

Now let us use the result (3.2) of not made measurement in the case of state (2.6).

One remaining unspecified item is $e^{-S_1\varphi_1}e^{S_2\varphi_2}$. Its G_3^+ expression follows, see e.g. [4], from known formulas. If

$$e^{I_{S_1}\varphi_1} = \alpha_1 + I_{S_1}\beta_1 = \alpha_1 + \beta_1 b_1^1 B_1 + \beta_1 b_1^2 B_2 + \beta_1 b_1^3 B_3$$

$$e^{I_{S_2}\varphi_2} = \alpha_2 + I_{S_2}\beta_2 = \alpha_2 + \beta_2 b_2^1 B_1 + \beta_2 b_2^2 B_2 + \beta_2 b_2^3 B_3$$

where

$$(\alpha_1)^2 + (\beta_1)^2\left((b_1^1)^2 + (b_1^2)^2 + (b_1^3)^2\right) = (\alpha_1)^2 + (\beta_1)^2 = 1$$

$$(\alpha_2)^2 + (\beta_2)^2\left((b_2^1)^2 + (b_2^2)^2 + (b_2^3)^2\right) = (\alpha_2)^2 + (\beta_2)^2 = 1$$

in some bivector basis $B_1B_2B_3 = 1$, with multiplication rules $B_1B_2 = -B_3$, $B_1B_3 = B_2$, $B_2B_3 = -B_1$, then the scalar part of $e^{-S_1\varphi_1}e^{S_2\varphi_2}$ is:

$$\alpha_1\alpha_2 - \beta_1\beta_2\left(b_1^1b_2^1 + b_1^2b_2^2 + b_1^3b_2^3\right),$$

and bivector part:

$$\begin{aligned}
&B_1\left(\beta_1\beta_2\left(b_1^3b_2^2 - b_1^2b_2^3\right) - \alpha_2\beta_1b_1^1 - \alpha_1\beta_2b_2^1\right)\\
&+B_2\left(\beta_1\beta_2\left(b_1^1b_2^3 - b_1^3b_2^1\right) - \alpha_2\beta_1b_1^2 - \alpha_1\beta_2b_2^2\right)\\
&+B_3\left(\beta_1\beta_2\left(b_1^2b_2^1 - b_1^1b_2^2\right) - \alpha_2\beta_1b_1^3 - \alpha_1\beta_2b_2^3\right)
\end{aligned}$$

Now it remains to apply (3.1) with

$$C_0^{Cl} = \alpha_1\alpha_2 - \beta_1\beta_2\left(b_1^1b_2^1 + b_1^2b_2^2 + b_1^3b_2^3\right)$$

$$C_1^{Cl} = \beta_1\beta_2\left(b_1^3b_2^2 - b_1^2b_2^3\right) - \alpha_2\beta_1b_1^1 - \alpha_1\beta_2b_2^1$$

$$C_2^{Cl} = \beta_1\beta_2\left(b_1^1b_2^3 - b_1^3b_2^1\right) - \alpha_2\beta_1b_1^2 - \alpha_1\beta_2b_2^2$$

$$C_3^{Cl} = \beta_1\beta_2\left(b_1^2b_2^1 - b_1^1b_2^2\right) - \alpha_2\beta_1b_1^3 - \alpha_1\beta_2b_2^3$$

The result of measuring of C^1 is, due to (3.1),

$$\begin{aligned}
C_m^1 = 4\cos^2\omega\left[(I_3I_S)\cdot r\right]&\Big[C_0^1 + C_3^1 I_S + \left(C_1^1\sin 2\omega t + C_2^1\cos 2\omega t\right)I_{B_0}\\
&+\left(C_2^1\sin 2\omega t - C_1^1\cos 2\omega t\right)I_{E_0}\Big]
\end{aligned}$$

And the result of measuring of $e^{-S_1\varphi_1}e^{S_2\varphi_2}$ is

$$\begin{aligned}
C_m^{Cl} = 4\cos^2\omega\left[(I_3I_S)\cdot r\right]&\Big[C_0^{Cl} + C_3^1 I_S + \left(C_1^{Cl}\sin 2\omega t + C_2^{Cl}\cos 2\omega t\right)I_{B_0}\\
&+\left(C_2^{Cl}\sin 2\omega t - C_1^{Cl}\cos 2\omega t\right)I_{E_0}\Big]
\end{aligned}$$

Finally, the result of not made measuring of C^2 is received from the result of measuring of C^1 as:

$$\frac{\left|C^2\right|}{\left|C^1\right|} C_m^1 C_m^{Cl},$$

where the product of two subalgebra G_3^+ elements is calculated by known formula (see, [4]):

$$g_1 g_2 = \left(\alpha_1 + I_{S_1}\beta_1\right)\left(\alpha_2 + I_{S_2}\beta_2\right) = \alpha_1\alpha_2 + I_{S_1}\alpha_2\beta_1 + I_{S_2}\alpha_1\beta_2 + I_{S_1}I_{S_2}\beta_1\beta_2$$

5. Conclusion

The geometric algebra lift of conventional quantum mechanics qubits is the game-changing quantum leap forward. The approach brings into reality a kind of physical field spreading through the whole three-dimensional space and values of the time parameter. All measured observable values are simultaneously available all together, not through looking one by one. In this way the new type of quantum computer appeared to be a kind of analog computer keeping and instantly processing information by and on sets of objects possessing an infinite number of degrees of freedom.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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