Journal of Applied Mathematics and Physics, 2023, 11, 448-456
https://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

Two Shaky Pillars of Quantum Computing

Alexander Soiguine

SOiGUINE Quantum Computing, Aliso Viejo, USA
Email: alex@soiguine.com

How to cite this paper: Soiguine, A. (2023) Two Shaky Pillars of Quantum Computing. *Journal of Applied Mathematics and Physics*, **11**, 448-456.
https://doi.org/10.4236/jamp.2023.112027

Received: January 4, 2023
Accepted: February 10, 2023
Published: February 13, 2023

Abstract

Superposition and entanglement are two theoretical pillars quantum computing rests upon. In the g-qubit theory quantum wave functions are identified by points on the surface of three-dimensional sphere $\mathbb{S}^3$. That gives different, more physically feasible explanation of what superposition and entanglement are. The core of quantum computing scheme should be in manipulation and transferring of wave functions on $\mathbb{S}^3$ as operators acting on observables and formulated in terms of geometrical algebra. In this way quantum computer will be a kind of analog computer keeping and processing information by sets of objects possessing infinite number of degrees of freedom, contrary to the two value bits or two-dimensional Hilbert space elements, qubits.

Keywords

Geometric Algebra, Wave Functions, Observables, Measurements

1. Introduction: Conventional Entanglement

Complementarity principle in physics says that a complete knowledge of phenomena on atomic dimensions requires a description of both wave and particle properties. The principle was announced in 1928 by the Danish physicist Niels Bohr. His statement was that depending on the experimental arrangement, the behavior of such phenomena as light and electrons is sometimes wavelike and sometimes particle-like and that it is impossible to observe both the wave and particle aspects simultaneously.

In the following it will be shown that actual weirdness of all conventional quantum mechanics comes from logical inconsistence of what is meant in basic quantum mechanical definitions and has nothing to do with the phenomena scale and the attached artificial complementarity principle [1] [2] [3] [4].

It will be explained below that theory should speak not about complementarity but about proper separation of measurement process arrangement into operator, three-sphere $\mathbb{S}^3$ element, acting on observable, and operand, measured observable.

It will be shown that quantum mechanics is not of something deeper but should be replaced by something conceptually different.

In the suggested alternative it is said that theory should speak not about complementarity but about proper dividing of the measurement process into operator, wave function, which is the three-sphere $\mathbb{S}^3$ element acting on observable, and operand, the measured observable.

A vector in quantum mechanics is the mathematical gadget used to describe the *state* of a quantum system, its status, what it's capable of doing. A state assigned to elementary particles there is given by a unit vector in a vector space, really a Hilbert space C^n, particularly C^2, encoding information about the state. The dimension n is the number of different observable things after making a measurement on the particle.

The simplest quantum mechanical state, qubit, reads:

$$C^2 \ni \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = z_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + z_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = z_1 |0\rangle + z_2 |1\rangle$$

It has just two observable "things" after measurement, say "up" for $|0\rangle$ and "down" for $|1\rangle$, with probabilities z_1^2 and z_2^2.

In the case of two particles vector space C^2 is generalized to density matrix defined on tensor product $C^2 \otimes C^2$ and in the case of N particles we get $C^2 \otimes C^2 \otimes \cdots \otimes C^2$, N-fold tensor product.

The appropriateness of tensor products is that the tensor product itself captures all ways that basic things can "interact" with each other.

2. Wave Functions in the g-Qubit Theory

Wave function will be a unit value element of even subalgebra of three-dimensional geometric algebra. Such elements will execute twisting of observables. Even subalgebra G_3^+ is subalgebra of elements of the form $M_3 = \alpha + I_S\beta$, where α and β are (real)[1] scalars and I_S is some unit bivector arbitrary placed in three-dimensional space.

Wave functions as elements of G_3^+ are naturally mapped onto unit sphere $\mathbb{S}^3$ [5] [6] [7].

If in some bivector basis $\{B_1, B_2, B_3\}$, with, for example, right-hand screw multiplication rules $B_1B_2B_3 = 1$, $B_1B_2 = -B_3$, $B_1B_3 = B_2$, $B_2B_3 = -B_1$, the twisting plane bivector is

$$I_S = b^1B_1 + b^2B_2 + b^3B_3,$$

then

$$\alpha + I_S\beta = \alpha + \beta b^1B_1 + \beta b^2B_2 + \beta b^3B_3$$

$$\alpha + I_S\beta \Rightarrow \{\alpha, \beta b^1, \beta b^2, \beta b^3\}$$

[1]In the current formalism scalars can only be real numbers. "Complex" scalars make no sense anymore.

and

$$\left(\alpha\right)^2+\left(\beta\right)^2\left(\left(b^1\right)^2+\left(b^2\right)^2+\left(b^3\right)^2\right)=\left(\alpha\right)^2+\left(\beta\right)^2=1,$$

since wave function is normalized and bivector I_S is a unit value one.

Wave function can always be conveniently written as exponent, see [7], Sec. 2. 5,

$$\alpha+I_S\beta=\mathrm{e}^{I_S\varphi},\quad \alpha=\cos\varphi,\quad \beta=\sin\varphi$$

The product of two exponents is again an exponent, because generally $|g_1g_2|=|g_1||g_2|$ and $\left|\mathrm{e}^{I_{S_1}\alpha}\mathrm{e}^{I_{S_2}\beta}\right|=\left|\mathrm{e}^{I_{S_1}\alpha}\right|\left|\mathrm{e}^{I_{S_2}\beta}\right|=1$.

Multiplication of an exponent by another exponent is often called *Clifford translation*. Using the term *translation* follows from the fact that Clifford translation does not change distances between the exponents it acts upon if we identify exponents as points on unit sphere $\mathbb{S}^3$:

$$\cos\alpha+I_S\sin\alpha=\cos\alpha+b_1\sin\alpha B_1+b_2\sin\alpha B_2+b_3\sin\alpha B_3$$
$$\Leftrightarrow\left\{\cos\alpha,b_1\sin\alpha,b_2\sin\alpha,b_3\sin\alpha\right\}$$

$$\left(\cos\alpha\right)^2+\left(b_1\sin\alpha\right)^2+\left(b_2\sin\alpha\right)^2+\left(b_3\sin\alpha\right)^2=1$$

This result follows again from $|g_1g_2|=|g_1||g_2|$:

$$\left|\mathrm{e}^{I_S\alpha}\left(g_1-g_2\right)\right|=\left|\mathrm{e}^{I_S\alpha}\right|\left|g_1-g_2\right|=\left|g_1-g_2\right|$$

Clifford translation of a wave function $\mathrm{e}^{I_{S_2}\varphi_2}$ *by* $\mathrm{e}^{I_{S_1}\varphi_1}$ is displacement of the wave function, point on $\mathbb{S}^3$, along big circle that is intersection of $\mathbb{S}^3$ *by* S_1 by parameter φ_1.

3. The Meaning of Schrodinger Equation

Let us take some vector $H(t)=I_3\left(\chi^1(t)B_1+\chi^2(t)B_2+\chi^3(t)B_3\right)$ and execute infinitesimal Clifford translation of a wave function $\mathrm{e}^{I_{S(t)}\varphi(t)}$ using bivector $-I_3H(t)$ and Clifford parameter $\left|H(t_0)\right|\Delta t$ at some instant of time t_0:

$$\mathrm{e}^{-I_3\frac{H(t_0)}{|H(t_0)|}|H(t_0)|\Delta t}\mathrm{e}^{I_{S(t_0)}\varphi(t_0)}$$

With denoting $I_3\dfrac{H(t_0)}{\left|H(t_0)\right|}\equiv I_H(t_0)$ we get:

$$\mathrm{e}^{I_{S(t_0+\Delta t)}\varphi(t_0+\Delta t)}\approx\mathrm{e}^{-I_H(t_0)|H(t_0)|\Delta t}\mathrm{e}^{I_{S(t_0)}\varphi(t_0)}$$

and

$$\lim_{\Delta t\to0}\frac{\mathrm{e}^{I_{S(t_0+\Delta t)}\varphi(t_0+\Delta t)}-\mathrm{e}^{I_{S(t_0)}\varphi(t_0)}}{\Delta t}$$
$$=\lim_{\Delta t\to0}\frac{\left(1-I_H(t_0)\left|H(t_0)\right|\Delta t\right)\mathrm{e}^{I_{S(t_0)}\varphi(t_0)}-\mathrm{e}^{I_{S(t_0)}\varphi(t_0)}}{\Delta t}$$
$$=-I_H(t_0)\left|H(t_0)\right|\mathrm{e}^{I_{S(t_0)}\varphi(t_0)}$$

That gives the Schrodinger equation:

$$-\frac{\partial}{\partial t}\mathrm{e}^{I_{S(t)}\varphi(t)} = I_H(t)\left|H(t)\right|\mathrm{e}^{I_{S(t)}\varphi(t)}$$

That means that the Schrodinger equation defines infinitesimal changes of wave functions under Clifford translations along big circles of $\mathbb{S}^3$.

4. Superposition of Two Basic Wave Functions Corresponding to $|0\rangle$ and $|1\rangle$

The quantum mechanical qubit state, $|\psi\rangle = z_1|0\rangle + z_2|1\rangle$, is linear combination of two basis states $|0\rangle$ and $|1\rangle$. In more details:

$$|\psi\rangle = \begin{pmatrix} \alpha + i\beta_1 \\ \beta_3 + i\beta_2 \end{pmatrix}$$

There exist infinite number of options to select triple $\{B_1, B_2, B_3\}$. Thus, the procedure of recovering a g-qubit associated with $|\psi\rangle = z_1|0\rangle + z_2|1\rangle$ is the following one:

It is necessary [6] [7] firstly, to define bivector B_{i_1} in three dimensions identifying the torsion plane. Secondly, choose another bivector B_{i_2} orthogonal to B_{i_1}. The third bivector B_{i_3}, orthogonal to both B_{i_1} and B_{i_1}, is then defined by the first two by orientation (handedness, right screw in the used case): $I_3 B_{i_1} I_3 B_{i_2} I_3 B_{i_3} = I_3$.

Wave functions in the suggested theory are operators acting through measurements on observables:

$$\left(\overline{\alpha + I_S\beta}\right) C\left(\alpha + I_S\beta\right)$$

For any wave function $\alpha + B_{i_1}\beta_i$, $i = 1,2,3$, corresponding to $|0\rangle$ (assuming $\alpha^2 + \beta_i^2 = 1$) we get:

$$\left(\alpha - B_{i_1}\beta_i\right) B_{i_1}\left(\alpha + B_{i_1}\beta_i\right) = \left(\alpha^2 + \beta_i^2\right) B_{i_1} = B_{i_1}$$

For the wave functions $\beta_{(i+2)\bmod 3} B_{(i+2)\bmod 3} + \beta_{(i+1)\bmod 3} B_{(i+1)\bmod 3}$,, $i = 1,2,3$, corresponding to $|1\rangle$ (with the agreement $3 \bmod 3 = 3$) the value of observable B_{i_1} is (with same assumption $\beta_{(i+2)\bmod 3}^2 + \beta_{(i+1)\bmod 3}^2 = 1$):

$$\left(-\beta_{(i+2)\bmod 3} B_{(i+2)\bmod 3} - \beta_{(i+1)\bmod 3} B_{(i+1)\bmod 3}\right) B_{i_1}\left(\beta_{(i+2)\bmod 3} B_{(i+2)\bmod 3} + \beta_{(i+1)\bmod 3} B_{(i+1)\bmod 3}\right)$$
$$= -\left(\beta_{(i+2)\bmod 3}^2 + \beta_{(i+1)\bmod 3}^2\right) B_{i_1} = -B_{i_1}$$

Let us take an arbitrary bivector observable expanded in basis $\{B_1, B_2, B_3\} \equiv \{B_{i_1}, B_{i_2}, B_{i_3}\}$:

$$C = C_1 B_1 + C_2 B_2 + C_3 B_3$$

The result of measurement by wave function corresponding to $|0\rangle$ is:

$$\begin{aligned} &\left(\alpha - \beta_1 B_1\right) C\left(\alpha + \beta_1 B_1\right) \\ &= C_1 B_1 + \left[C_2\left(\alpha^2 - \beta_1^2\right) - 2C_3\alpha\beta_1\right] B_2 + \left[C_3\left(\alpha^2 - \beta_1^2\right) + 2C_2\alpha\beta_1\right] B_3 \\ &= C_1 B_1 + \left(C_2\cos 2\varphi - C_3\sin 2\varphi\right) B_2 + \left(C_2\sin 2\varphi + C_3\cos 2\varphi\right) B_3, \end{aligned} \quad (4.1)$$

using parametrization $\alpha = \cos\varphi$, $\beta_1 = \sin\varphi$.

The result of measurement by wave function corresponding to $|1\rangle$ is:

$$\begin{aligned}&\left(-\beta_2 B_2 - \beta_3 B_3\right) C \left(\beta_2 B_2 + \beta_3 B_3\right)\\&= -C_1 B_1 + \left[C_2\left(\beta_2^2 - \beta_3^2\right) + 2C_3\beta_2\beta_3\right] B_2 + \left[2C_2\beta_2\beta_3 - C_3\left(\beta_2^2 - \beta_3^2\right)\right] B_3\\&= -C_1 B_1 + \left(C_2 \cos 2\theta + C_3 \sin 2\theta\right) B_2 + \left(C_2 \sin 2\theta - C_3 \cos 2\theta\right) B_3,\end{aligned} \tag{4.2}$$

with $\beta_2 = \cos\theta$, $\beta_3 = \sin\theta$.

This is a deeper result compared with conventional quantum mechanics where

$$|0\rangle = \begin{pmatrix} \alpha + i\beta_1 \\ 0 \end{pmatrix} \text{ and } |1\rangle = \begin{pmatrix} 0 \\ \beta_3 + i\beta_2 \end{pmatrix}$$

Conclusion:

- Measurement of observable $C = C_1 B_1 + C_2 B_2 + C_3 B_3$ by any wave function corresponding to $|0\rangle$ is bivector with the B_1 component equal to unchanged value C_1. The B_2 and B_3 components of the result of measurement are equal to B_2 and B_3 components of C rotated by angle 2φ defined by $\alpha = \cos\varphi$, $\beta_1 = \sin\varphi$ where plane of rotation is B_1.
- Measurement of observable $C = C_1 B_1 + C_2 B_2 + C_3 B_3$ by any wave function corresponding to $|1\rangle$ is bivector with the B_1 component equal to flipped value $-C_1$. The B_2 and B_3 components of the result of measurement are equal to B_2 and B_3 components of C rotated by angle 2θ defined by $\beta_2 = \cos\theta$, $\beta_3 = \sin\theta$ where plane of rotation is B_1 but direction of rotation is opposite to the case of $|0\rangle$.

If we denote by $so_{|0\rangle}$ and $so_{|1\rangle}$ arbitrary wave functions represented in G_3^+ by $\alpha + \beta_1 B_1$ and $\beta_2 B_2 + \beta_3 B_3 = \left(\beta_3 + \beta_2 B_1\right) B_3$ they only differ by factor B_3 in $so_{|1\rangle}$, thus for the measurement by them we have:

$$\overline{so_{|1\rangle}} C so_{|1\rangle} = \overline{B_3}\,\overline{so_{|0\rangle}} C so_{|0\rangle} B_3$$

That simply means that the measurement on the left side is received from $\overline{so_{|0\rangle}} C so_{|0\rangle}$ by its flipping in plane B_3.

Probabilities of the results of measurements are measures of wave functions on $\mathbb{S}^3$ surface giving considered results.

Suppose we are interested in the probability of the result of measurement in which the observable component $C_1 B_1$ does not change. This is relative measure of wave functions $\sqrt{\alpha^2 + \beta_1^2}\left(\frac{\alpha}{\sqrt{\alpha^2 + \beta_1^2}} + \frac{\beta_1}{\sqrt{\alpha^2 + \beta_1^2}} B_1\right)$ in the measurements:

$$\sqrt{\alpha^2 + \beta_1^2}\left(\frac{\alpha}{\sqrt{\alpha^2 + \beta_1^2}} - \frac{\beta_1}{\sqrt{\alpha^2 + \beta_1^2}} B_1\right) C \sqrt{\alpha^2 + \beta_1^2}\left(\frac{\alpha}{\sqrt{\alpha^2 + \beta_1^2}} + \frac{\beta_1}{\sqrt{\alpha^2 + \beta_1^2}} B_1\right) \tag{4.3}$$

That measure is equal to $\alpha^2 + \beta_1^2$, that is equal to z_1^2 in the down mapping from G_3^+ to Hilbert space of $z_1|0\rangle + z_2|1\rangle$. Thus, we have clear explanation of common quantum mechanics wisdom on "probability of finding system in state $|0\rangle$".

Similar calculations explain correspondence of $\beta_3^2+\beta_2^2$ to z_2^2 in the qubit $z_1|0\rangle+z_2|1\rangle$ when the component C_1B_1 in measurement just got flipped.

Let us consider superposition of $\alpha+\beta_1B_1$ and $\beta_2B_2+\beta_3B_3$ with some coefficients p_1 and p_2,

$$p_1(\alpha+\beta_1B_1)+p_2(\beta_2B_2+\beta_3B_3),$$

and measuring by it of $C=C_1B_1+C_2B_2+C_3B_3$.

$$\begin{aligned}&\left[p_1(\alpha-\beta_1B_1)+p_2(-\beta_2B_2-\beta_3B_3)\right]C\left[p_1(\alpha+\beta_1B_1)+p_2(\beta_2B_2+\beta_3B_3)\right]\\&=p_1(\alpha-\beta_1B_1)Cp_1(\alpha+\beta_1B_1)+p_2(-\beta_2B_2-\beta_3B_3)Cp_2(\beta_2B_2+\beta_3B_3)\\&\quad+p_2(-\beta_2B_2-\beta_3B_3)Cp_1(\alpha+\beta_1B_1)+p_1(\alpha-\beta_1B_1)Cp_2(\beta_2B_2+\beta_3B_3)\\&=p_1(\alpha-\beta_1B_1)Cp_1(\alpha+\beta_1B_1)+p_2(-\beta_2B_2-\beta_3B_3)Cp_2(\beta_2B_2+\beta_3B_3)\\&\quad+p_1(\alpha-\beta_1B_1)Cp_1(\alpha+\beta_1B_1)p_1(\alpha-\beta_1B_1)p_2(\beta_2B_2+\beta_3B_3)\\&\quad+p_2(-\beta_2B_2-\beta_3B_3)Cp_2(\beta_2B_2+\beta_3B_3)p_2(-\beta_2B_2-\beta_3B_3)p_1(\alpha+\beta_1B_1)\end{aligned}$$

It follows from this formula that the result of measurement by wave function $p_1(\alpha+\beta_1B_1)+p_2(\beta_2B_2+\beta_3B_3)$ makes the C_1B_1 component unchanged and two other components rotated around the normal to B_1, see (4.1) and (4.3), with probability p_1^2 (item $p_1(\alpha-\beta_1B_1)Cp_1(\alpha+\beta_1B_1)$). Then it just flips the C_1B_1 component and two other components rotated around the normal to B_1, but in opposite direction see (4.2) with probability p_2^2 (item $p_2(-\beta_2B_2-\beta_3B_3)Cp_2(\beta_2B_2+\beta_3B_3)$).

Other two items are correspondingly the first above item subjected to Clifford (parallel) translation on $\mathbb{S}^3$ by $p_1p_2(\alpha-\beta_1B_1)(\beta_2B_2+\beta_3B_3)$ and the second item subjected to opposite Clifford translation $p_1p_2(-\beta_2B_2-\beta_3B_3)(\alpha+\beta_1B_1)$. They are neither (4.1) nor (4.2) and their probabilities to make C_1B_1 unchanged or flipped are zero. Thus, they give two other different available measurement results.

5. Superposition of Two Arbitrary Wave Functions

Any arbitrary G_3^+ wave function $\alpha+\beta_1B_1+\beta_2B_2+\beta_3B_3$ can be rewritten either as 0-type wave function or 1-type wave function:

$$\alpha+\beta_1B_1+\beta_2B_2+\beta_3B_3=\alpha+I_{S(\beta_1,\beta_2,\beta_3)}\sqrt{\beta_1^2+\beta_2^2+\beta_3^2},$$

where $I_{S(\beta_1,\beta_2,\beta_3)}=\dfrac{\beta_1B_1+\beta_2B_2+\beta_3B_3}{\sqrt{\beta_1^2+\beta_2^2+\beta_3^2}}$, 0-type,

or

$$\begin{aligned}\alpha+\beta_1B_1+\beta_2B_2+\beta_3B_3&=(\beta_3+\beta_2B_1-\beta_1B_2-\alpha B_3)B_3\\&=\left(\beta_3+I_{S(\beta_2,-\beta_1,-\alpha)}\sqrt{\alpha^2+\beta_1^2+\beta_2^2}\right)B_3,\end{aligned}$$

where $I_{S(\beta_2,-\beta_1,-\alpha)}=\dfrac{\beta_2B_1-\beta_1B_2-\alpha B_3}{\sqrt{\alpha^2+\beta_1^2+\beta_2^2}}$, 1-type.

All that means that any G_3^+ wave function $\alpha+\beta_1B_1+\beta_2B_2+\beta_3B_3$ measuring observable $C_1B_1+C_2B_2+C_3B_3$ does not change the observable projection onto plane of $I_{S(\beta_1,\beta_2,\beta_3)}=\frac{\beta_1B_1+\beta_2B_2+\beta_3B_3}{\sqrt{\beta_1^2+\beta_2^2+\beta_3^2}}$ and just flips the observable projection onto plane $I_{S(\beta_2,-\beta_1,-\alpha)}=\frac{\beta_2B_1-\beta_1B_2-\alpha B_3}{\sqrt{\alpha^2+\beta_1^2+\beta_2^2}}$.

Take two arbitrary wave functions and rewrite the first one as 0-type wave function and the second one as 1-type wave function. Then all the results of Sec. 2 become applicable for their superposition. It will follow that there will be a result of measurement

$$p_1^2\left(\alpha-I_{S(\beta_1,\beta_2,\beta_3)}\sqrt{\beta_1^2+\beta_2^2+\beta_3^2}\right)C\left(\alpha+I_{S(\beta_1,\beta_2,\beta_3)}\sqrt{\beta_1^2+\beta_2^2+\beta_3^2}\right)$$

not changing the projection of C onto plane of $I_{S(\beta_1,\beta_2,\beta_3)}$ and keeping probability p_1^2; plus, result of measurement

$$p_2^2\left(-B_3\left(\beta_3-I_{S(\beta_2,-\beta_1,-\alpha)}\sqrt{\alpha^2+\beta_1^2+\beta_2^2}\right)\right)C\left(\left(\beta_3+I_{S(\beta_2,-\beta_1,-\alpha)}\sqrt{\alpha^2+\beta_1^2+\beta_2^2}\right)B_3\right)$$

just flipping projection of C in plane of $I_{S(\beta_2,-\beta_1,-\alpha)}$ and keeping probability p_2^2. Two other results represent the first two subjected to Clifford (parallel) translations on the sphere $\mathbb{S}^3$ by

$$p_1p_2\left(\alpha-I_{S(\beta_1,\beta_2,\beta_3)}\sqrt{\beta_1^2+\beta_2^2+\beta_3^2}\right)\left(\beta_3+I_{S(\beta_2,-\beta_1,-\alpha)}\sqrt{\alpha^2+\beta_1^2+\beta_2^2}\right)$$

and

$$p_1p_2\left(\beta_3-I_{S(\beta_2,-\beta_1,-\alpha)}\sqrt{\alpha^2+\beta_1^2+\beta_2^2}\right)\left(\alpha+I_{S(\beta_1,\beta_2,\beta_3)}\sqrt{\beta_1^2+\beta_2^2+\beta_3^2}\right)$$

correspondingly.

6. Entanglement in Measurements

Whilst the Schrodinger equation governs infinitesimal transformations of a wave function by Clifford translations a finite Clifford translation moves a wave function along a big circle of $\mathbb{S}^3$ by any Clifford parameter.

In G_3^+ multiplication is:

$$g_1g_2=\left(\alpha_1+I_{S_1}\beta_1\right)\left(\alpha_2+I_{S_2}\beta_2\right)=\alpha_1\alpha_2+I_{S_1}\alpha_2\beta_1+I_{S_2}\alpha_1\beta_2+I_{S_1}I_{S_2}\beta_1\beta_2$$

It is not commutative due to the not commutative product of bivectors $I_{S_1}I_{S_2}$. Indeed, taking vectors to which I_{S_1} and I_{S_2} are dual: $s_1=-I_3I_{S_1}$, $s_2=-I_3I_{S_2}$, we have, see [7], Sec. 1.1:

$$I_{S_1}I_{S_2}=-s_1\cdot s_2-I_3\left(s_1\times s_2\right)$$

Then:

$$g_1g_2=\alpha_1\alpha_2-\left(s_1\cdot s_2\right)\beta_1\beta_2+I_{S_1}\alpha_2\beta_1+I_{S_2}\alpha_1\beta_2-I_3\left(s_1\times s_2\right)\beta_1\beta_2$$

and

$$g_2g_1=\alpha_1\alpha_2-\left(s_1\cdot s_2\right)\beta_1\beta_2+I_{S_1}\alpha_2\beta_1+I_{S_2}\alpha_1\beta_2+I_3\left(s_1\times s_2\right)\beta_1\beta_2$$

I the case when both elements are of exponent form:

$$e^{I_{S_1}\varphi_1} = \alpha_1 + I_{S_1}\beta_1 = \alpha_1 + \beta_1 b_1^1 B_1 + \beta_1 b_1^2 B_2 + \beta_1 b_1^3 B_3$$

$$e^{I_{S_2}\varphi_2} = \alpha_2 + I_{S_2}\beta_2 = \alpha_2 + \beta_2 b_2^1 B_1 + \beta_2 b_2^2 B_2 + \beta_2 b_2^3 B_3,$$

with

$$(\alpha_1)^2 + (\beta_1)^2 \left((b_1^1)^2 + (b_1^2)^2 + (b_1^3)^2 \right) = (\alpha_1)^2 + (\beta_1)^2 = 1$$

$$(\alpha_2)^2 + (\beta_2)^2 \left((b_2^1)^2 + (b_2^2)^2 + (b_2^3)^2 \right) = (\alpha_2)^2 + (\beta_2)^2 = 1,$$

as in the case a wave function and Clifford translation, we get:

$$\begin{aligned} e^{I_{S_2}\varphi_2} e^{I_{S_1}\varphi_1} &= \cos\varphi_1 \cos\varphi_2 + (s_1 \cdot s_2)\sin\varphi_1 \sin\varphi_2 + I_3 s_2 \cos\varphi_1 \sin\varphi_2 \\ &+ I_3 s_1 \cos\varphi_2 \sin\varphi_1 - I_3 (s_2 \times s_1)\sin\varphi_1 \sin\varphi_2 \end{aligned}$$

Then it follows that two wave functions are, in any case, connected by the Clifford translation:

$$e^{I_{S_2}\varphi_2} = \left(e^{I_{S_2}\varphi_2} e^{-I_{S_1}\varphi_1} \right) e^{I_{S_1}\varphi_1} \equiv Cl(S_2, \varphi_2, S_1, \varphi_1) e^{I_{S_1}\varphi_1},$$

where

$$\begin{aligned} Cl(S_2, \varphi_2, S_1, \varphi_1) &\equiv e^{I_{S_2}\varphi_2} e^{-I_{S_1}\varphi_1} \\ &= \cos\varphi_1 \cos\varphi_2 + (s_1 \cdot s_2)\sin\varphi_1 \sin\varphi_2 + I_3 s_2 \cos\varphi_1 \sin\varphi_2 \\ &+ I_3 s_1 \cos\varphi_2 \sin\varphi_1 + I_3 (s_2 \times s_1)\sin\varphi_1 \sin\varphi_2 \end{aligned}.$$

From knowing Clifford translation connecting any two wave functions as points on $\mathbb{S}^3$ it follows that the result of measurement of any observable C by wave function $e^{I_{S_1}\varphi_1}$, for example $e^{-I_{S_1}\varphi_1} C e^{I_{S_1}\varphi_1} \equiv C(S_1, \varphi_1)$, immediately gives the result of (not made) measurement by $e^{I_{S_2}\varphi_2}$:

$$\begin{aligned} e^{-I_{S_2}\varphi_2} C e^{I_{S_2}\varphi_2} &= e^{-I_{S_2}\varphi_2} e^{I_{S_1}\varphi_1} e^{-I_{S_1}\varphi_1} C e^{I_{S_1}\varphi_1} e^{-I_{S_1}\varphi_1} e^{I_{S_2}\varphi_2} \\ &= e^{-I_{S_2}\varphi_2} e^{I_{S_1}\varphi_1} C(S_1, \varphi_1) e^{-I_{S_1}\varphi_1} e^{I_{S_2}\varphi_2} \\ &= Cl(S_2, -\varphi_2, S_1, -\varphi_1) C(S_1, \varphi_1) \overline{Cl(S_2, -\varphi_2, S_1, -\varphi_1)} \end{aligned}$$

When assuming observables are also identified by points on $\mathbb{S}^3$ and thus are connected by formulas as the above one we get that the measurements of any amount of observables by arbitrary set of wave functions are simultaneously available.

7. Conclusion

The suggested formalism gives different, more physically feasible explanation of what is superposition and entanglement. Superposition of any two wave functions in the frame of g-qubit theory gives another wave function the result of measurement by which is more complicated than in conventional quantum mechanics. In addition to the two results of measurements coming from composed items of the wave functions there appear two additional items which are Clifford (parallel) translations of the first two results in opposite directions on the sphere $\mathbb{S}^3$. The core of quantum computing scheme should be in manipulation and transferring of wave functions on $\mathbb{S}^3$ as operators acting on observables and formulated in terms of geometrical algebra.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Doran, C.L.A. (2003) Geometric Algebra for Physicists. Cambridge University Press, Cambridge. https://doi.org/10.1017/CBO9780511807497

[2] Hestenes, D. and Sobczyk, G. (1984) Clifford Algebra to Geometric Calculus: A Unified Language for Mathematics and Physics. Springer Dordrecht, Berlin. https://doi.org/10.1007/978-94-009-6292-7

[3] Pachos, J. (2012) Introduction to Topological Quantum Computation. Cambridge University Press, Cambridge. https://doi.org/10.1017/CBO9780511792908

[4] Gallier, J. (2011) Geometric Methods and Applications. Springer New York, NY. https://doi.org/10.1007/978-1-4419-9961-0

[5] Soiguine, A. (2014) What Quantum State Really Is? http://arxiv.org/abs/1406.3751

[6] Soiguine, A. (2015) Geometric Algebra, Qubits, Geometric Evolution, and All That. http://arxiv.org/abs/1502.02169

[7] Soiguine, A. (2020) The Geometric Algebra Lift of Qubits and Beyond. LAMBERT Academic Publishing, Riga.