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The 3-Sphere Instead of Hilbert Space

Alexander Soiguine

SOiGUINE Quantum Computing, Aliso Viejo, USA
Email: alex@soiguine.com

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Abstract

The Geometric Algebra formalism opens the door to developing a theory replacing conventional quantum mechanics. Generalizations, stemming from changing of complex numbers by geometrically feasible objects in three dimensions, followed by unambiguous definition of states, observables, measurements, bring into reality clear explanations of weird quantum mechanical features, for example primitively considering atoms as a kind of solar system. The three-sphere $\mathbb{S}^3$ becomes the playground of the torsion kind states eliminating abstract Hilbert space vectors. The $\mathbb{S}^3$ points evolve, governed by updated Schrodinger equation, and act in measurements on observable as operators.

Keywords

Geometric Algebra, States, Observables, Measurements

1. Introduction. States, Observables, Measurements

Complementarity principle in physics says that a complete knowledge of phenomena on atomic dimensions requires a description of both wave and particle properties. The principle was announced in 1928 by the Danish physicist Niels Bohr. His statement was that depending on the experimental arrangement, the behavior of such phenomena as light and electrons is sometimes wavelike and sometimes particle-like and that it is impossible to observe both the wave and particle aspects simultaneously.

In the following it will be shown that actual weirdness of all conventional quantum mechanics comes from logical inconsistence of what is meant in basic quantum mechanical definitions and has nothing to do with the phenomena scale and the attached artificial complementarity principle.

It will be explained below that theory should speak not about complementarity but about proper separation of measurement process arrangement into operator, three-sphere $\mathbb{S}^3$ element, acting on observable, and operand, measured observable.

General Definitions

Unambiguous definition of states and observables, does not matter are we in "classical" or "quantum" frame, should follow the general paradigm [1] [2] [3]:

- Measurement of observable $O(\mu)$ by state[1] $S(\lambda)$ is a map:

$$\big(S(\lambda), O(\mu)\big) \to O(\nu),$$

where $O(\mu)$ is an element of the set of observables, $S(\lambda)$ is element of, generally though not necessarily, another set, set of states.

- The result (value) of a measurement of observable $O(\mu)$ by state $S(\lambda)$ is the result of a map sequence:

$$\big(S(\lambda), O(\mu)\big) \to O(\nu) \to V(B),$$

where V is a set of (Boolean) algebra subsets identifying possible results of measurements.

Thus, state and observable are different things. Evolution of a state should be considered separately, and then action of modified state will be applied to observable in measurement.

The option to expand, to lift the space where physical processes are considered, may have critical consequences to a theory. A kind of expanding is the core of the suggested formulation aimed at the theory deeper than conventional quantum mechanics. States as Hilbert space complex valued vectors with formal imaginary unit are lifted to a torsion kind object identified by points on sphere $\mathbb{S}^3$.

2. Working with G-Qubits Instead of Qubits

A theory that is an alternative to conventional quantum mechanics has been under development for a while, see [1] [2] [4] [5].

Its novel features are:

- Replacing complex numbers by elements of even subalgebra of geometric algebra in three dimensions, that's by elements of the form "scalar plus bivector".
- Elementary physical objects bear the structure: position in space plus explicitly defined object as the G_3, geometric algebra in three dimensions, elements.
- Operators acting on those objects are identified as direct sums of position translation and points on the three-sphere $\mathbb{S}^3$. All those points are connected, due to hedgehog theorem, by parallel (Clifford) translations.
- Evolution of the $\mathbb{S}^3$ part of operators by Clifford translations is governed by generalization of the Schrodinger equation with unit bivectors in three dimensions instead of formal imaginary unit.

In the following the $\mathbb{S}^3$ part of the operators will only be considered.

[1]One should say "by a state". State is operator acting on observable.

Qubits, identifying states in conventional quantum mechanics, mathematically are elements of the two-dimensional complex spaces, namely $\begin{pmatrix} x_1 + iy_1 \\ x_2 + iy_2 \end{pmatrix}$, conditioned by $\|x_1 + iy_1\|^2 + \|x_2 + iy_2\|^2 = 1$, that is unit value elements of the C^2, Hilbert space.

Imaginary unit i is used formally with the property $i^2 = -1$. In another accepted notations a qubit is:

$$C^2 \ni \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = z_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + z_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = z_1 |0\rangle + z_2 |1\rangle$$

In the suggested formalism complex numbers $x + iy$ are replaced with elements of even subalgebra of G_3—geometric algebra in three dimensions.

Even subalgebra G_3^+ is subalgebra of elements of the form $M_3 = \alpha + I_S \beta$, where α and β are (real)[2] scalars and I_S is some unit bivector arbitrary placed in three-dimensional space. Elements of G_3^+ can be depict as in **Figure 1**.

In the G_3^+ multiplication is more complicated than in Hilbert space C^2. It reads:

$$g_1 g_2 = (\alpha_1 + I_{S_1} \beta_1)(\alpha_2 + I_{S_2} \beta_2) = \alpha_1 \alpha_2 + I_{S_1} \alpha_2 \beta_1 + I_{S_2} \alpha_1 \beta_2 + I_{S_1} I_{S_2} \beta_1 \beta_2$$

It is not commutative due to the not commutative product of bivectors $I_{S_1} I_{S_2}$. Indeed, taking vectors to which I_{S_1} and I_{S_2} are dual: $s_1 = -I_3 I_{S_1}$, $s_2 = -I_3 I_{S_2}$, we have:

$$I_{S_1} I_{S_2} = -s_1 \cdot s_2 - I_3 (s_1 \times s_2), \quad I_3 \text{ is oriented unit value volume.}$$

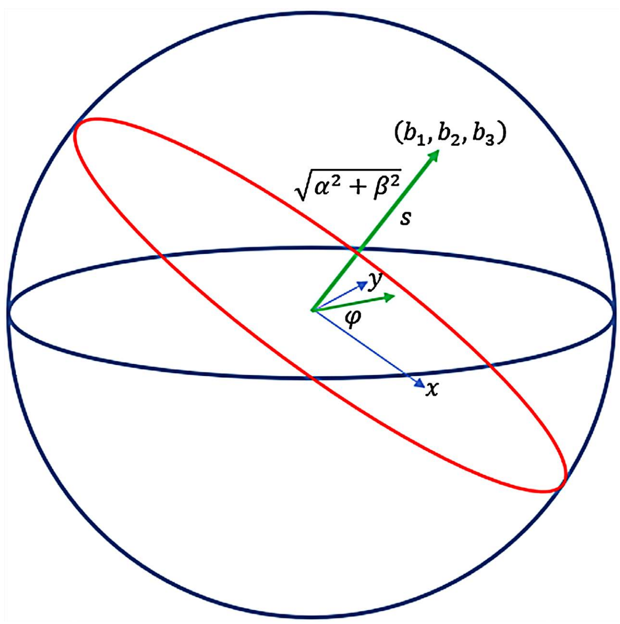

Figure 1. An element of G_3^+.

[2]In the current formalism scalars can only be real numbers. "Complex" scalars make no sense anymore, see, for example [2] [5].

Then:

$$g_1g_2 = \alpha_1\alpha_2 - (s_1 \cdot s_2)\beta_1\beta_2 + I_{S_1}\alpha_2\beta_1 + I_{S_2}\alpha_1\beta_2 - I_3(s_1 \times s_2)\beta_1\beta_2$$

and

$$g_2g_1 = \alpha_1\alpha_2 - (s_1 \cdot s_2)\beta_1\beta_2 + I_{S_1}\alpha_2\beta_1 + I_{S_2}\alpha_1\beta_2 + I_3(s_1 \times s_2)\beta_1\beta_2$$

We see that if $I_{S_1} = I_{S_2} = I_S$ then $s_1 \cdot s_2 = 1$, $s_1 \times s_2 = 0$, so

$$g_1g_2 = g_2g_1 = \alpha_1\alpha_2 - \beta_1\beta_2 + I_S(\alpha_2\beta_1 + \alpha_1\beta_2)$$

that is the same, up to replacing i by I_S, as for complex numbers.

Unit value elements of G_3^+, when $\alpha^2 + \beta^2 = 1$, will be called *g-qubits.* The wave functions, states, implemented as g-qubits store much more information than qubits, see **Figure 2**.

3. Lift of Qubits to G-Qubits

3.1. Lift of Quantum Mechanical Qubit States to G-Qubits

Take right-hand screw oriented basis $\{B_1, B_2, B_3\}$ of unit value bivectors, with the multiplication rules $B_1B_2 = -B_3$, $B_1B_3 = B_2$, $B_2B_3 = -B_1$, $I_3B_1I_3B_2I_3B_3 = I_3$ (or equivalently $B_1B_2B_3 = 1$), where I_3 is oriented unit value volume, pseudoscalar, in three dimensions, see **Figure 3**.

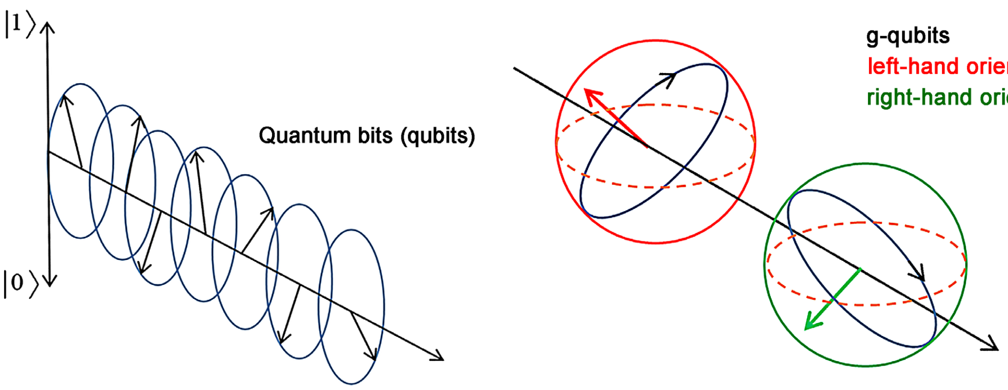

Figure 2. Geomectrically picted qubits and g-qubits.

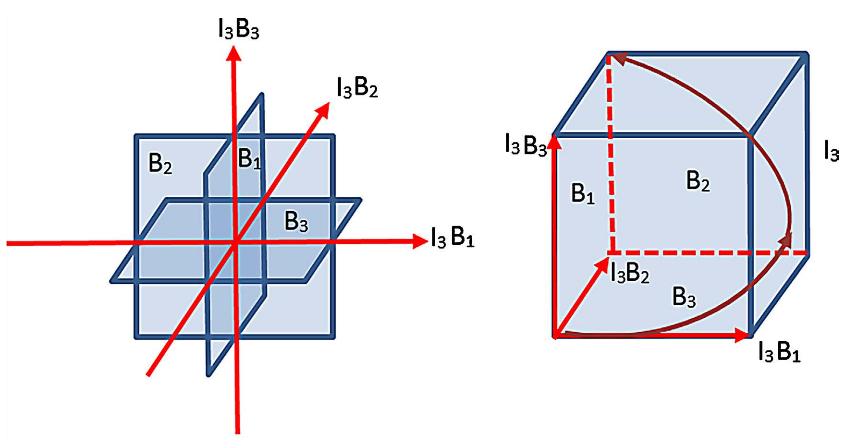

Figure 3. Basis of bivectors, dual vectors and unit value pseudoscalar.

The quantum mechanical qubit state, $|\psi\rangle = z_1|0\rangle + z_2|1\rangle$, is linear combination of two basis states $|0\rangle$ and $|1\rangle$. In the G_3^+ terms these two states correspond to the following classes of equivalence in G_3^+, depending particularly on which basis bivector is selected as torsion plane:

- If B_1 is taken as torsion plane, then
  - State $|0\rangle$ has fiber (level set) of the G_3^+ elements $so(\alpha,\beta,S)_{|0\rangle}$ (0-type G_3^+ states):

$$\alpha + \beta_1 B_1,\ \ \alpha^2 + \beta_1^2 = 1$$

  - State $|1\rangle$ has fiber of the G_3^+ elements $so(\alpha,\beta,S)_{|1\rangle}$ (1-type G_3^+ states):

$$\beta_3 B_3 + \beta_2 B_2 = (\beta_3 + \beta_2 B_1) B_3,\ \ \beta_3^2 + \beta_2^2 = 1$$

- If B_2 is taken as torsion plane, then
  - State $|0\rangle$ has fiber (level set) of the G_3^+ elements $so(\alpha,\beta,S)_{|0\rangle}$ (0-type G_3^+ states):

$$\alpha + \beta_2 B_2,\ \ \alpha^2 + \beta_2^2 = 1$$

  - State $|1\rangle$ has fiber of the G_3^+ elements $so(\alpha,\beta,S)_{|0\rangle}$ (1-type G_3^+ states):

$$\beta_1 B_1 + \beta_3 B_3 = (\beta_1 + \beta_3 B_2) B_1,\ \ \beta_1^2 + \beta_3^2 = 1$$

- If B_3 is taken as torsion plane, then
  - State $|0\rangle$ has fiber (level set) of the G_3^+ elements $so(\alpha,\beta,S)_{|0\rangle}$ (0-type G_3^+ states):

$$\alpha + \beta_3 B_3,\ \ \alpha^2 + \beta_3^2 = 1$$

  - State $|1\rangle$ has fiber of the G_3^+ elements $so(\alpha,\beta,S)_{|0\rangle}$ (1-type G_3^+ states):

$$\beta_1 B_1 + \beta_2 B_2 = (\beta_2 + \beta_1 B_3) B_2,\ \ \beta_2^2 + \beta_1^2 = 1$$

3.2. Implementation of Definitions 1.1 in the G-Qubit State Case

General definition of measurement in the suggested approach is based on:

- the set of observables, particularly elements of G_3^+,
- the set of states, normalized elements of G_3^+, g-qubits,
- special case of measurement of a G_3^+ observable $C = C_0 + C_1 B_1 + C_2 B_2 + C_3 B_3$ by g-qubit (wave function) $\alpha + I_S\beta = \alpha + \beta_1 B_1 + \beta_2 B_2 + \beta_3 B_3$ is defined as

$$(\alpha - I_S\beta) C (\alpha + I_S\beta)$$

with the result:

$$\begin{aligned}
&C_0 + C_1 B_1 + C_2 B_2 + C_3 B_3 \xrightarrow{\alpha + \beta_1 B_1 + \beta_2 B_2 + \beta_3 B_3} \\
&C_0 + \Big(C_1\big[(\alpha^2 + \beta_1^2) - (\beta_2^2 + \beta_3^2)\big] + 2C_2(\beta_1\beta_2 - \alpha\beta_3) + 2C_3(\alpha\beta_2 + \beta_1\beta_3)\Big) B_1 \\
&+ \Big(2C_1(\alpha\beta_3 + \beta_1\beta_2) + C_2\big[(\alpha^2 + \beta_2^2) - (\beta_1^2 + \beta_3^2)\big] + 2C_3(\beta_2\beta_3 - \alpha\beta_1)\Big) B_2 \\
&+ \Big(2C_1(\beta_1\beta_3 - \alpha\beta_2) + 2C_2(\alpha\beta_1 + \beta_2\beta_3) + C_3\big[(\alpha^2 + \beta_3^2) - (\beta_1^2 + \beta_2^2)\big]\Big) B_3
\end{aligned} \tag{3.1}$$

Since g-qubit (state, wave function) is normalized, the measurement can be written in exponential form:

$$e^{-I_S\varphi} C e^{I_S\varphi}$$

where $\varphi = \cos^{-1}\alpha$. The above is updated variant of quantum mechanical formula $\langle\psi|A|\psi\rangle$

The lift from C^2 to G_3^+ needs a $\{B_1, B_2, B_3\}$ reference frame of unit value bivectors. This frame, as a solid, can be arbitrary rotated in three dimensions. In that sense we have principal fiber bundle $G_3^+ \to C^2$ with the standard fiber as group of rotations which is also effectively identified by elements of G_3^+. Probabilities of the results of measurements are measures of the $\mathbb{S}^3$ states giving considered results.

Suppose we are interested in the probability of the result of measurement in which the observable component $C_1 B_1$ does not change. This is relative measure of states $\sqrt{\alpha^2+\beta_1^2}\left(\frac{\alpha}{\sqrt{\alpha^2+\beta_1^2}} + \frac{\beta_1}{\sqrt{\alpha^2+\beta_1^2}} B_1\right)$ in the measurements:

$$\sqrt{\alpha^2+\beta_1^2}\left(\frac{\alpha}{\sqrt{\alpha^2+\beta_1^2}} - \frac{\beta_1}{\sqrt{\alpha^2+\beta_1^2}} B_1\right) C \sqrt{\alpha^2+\beta_1^2}\left(\frac{\alpha}{\sqrt{\alpha^2+\beta_1^2}} + \frac{\beta_1}{\sqrt{\alpha^2+\beta_1^2}} B_1\right)$$

That measure is equal to $\alpha^2+\beta_1^2$, that is equal to z_1^2 in the down mapping from G_3^+ to Hilbert space of $z_1|0\rangle + z_2|1\rangle$. Thus, we have clear explanation of common quantum mechanics wisdom on "probability of finding system in state $|0\rangle$".

Similar calculations explain correspondence of $\beta_3^2+\beta_2^2$ to z_2^2 in the qubit $z_1|0\rangle + z_2|1\rangle$ when the component $C_1 B_1$ in measurement just got flipped.

Any arbitrary G_3^+ state $so(\alpha,\beta,S) = \alpha + \beta_1 B_1 + \beta_2 B_2 + \beta_3 B_3$ can be rewritten either as 0-type state or 1-type state:

$$\alpha + \beta_1 B_1 + \beta_2 B_2 + \beta_3 B_3 = \alpha + I_{S(\beta_1,\beta_2,\beta_3)}\sqrt{\beta_1^2+\beta_2^2+\beta_3^2},$$

where $I_{S(\beta_1,\beta_2,\beta_3)} = \frac{\beta_1 B_1 + \beta_2 B_2 + \beta_3 B_3}{\sqrt{\beta_1^2+\beta_2^2+\beta_3^2}}$, 0-type,

or

$$\begin{aligned}\alpha + \beta_1 B_1 + \beta_2 B_2 + \beta_3 B_3 &= \left(\beta_3 + \beta_2 B_1 - \beta_1 B_2 - \alpha B_3\right) B_3 \\ &= \left(\beta_3 + I_{S(\beta_2,-\beta_1,-\alpha)}\sqrt{\alpha^2+\beta_1^2+\beta_2^2}\right) B_3\end{aligned}$$

where $I_{S(\beta_2,-\beta_1,-\alpha)} = \frac{\beta_2 B_1 - \beta_1 B_2 - \alpha B_3}{\sqrt{\alpha^2+\beta_1^2+\beta_2^2}}$, 1-type.

All that means that any G_3^+ state $\alpha + \beta_1 B_1 + \beta_2 B_2 + \beta_3 B_3$ measuring observable $C_1 B_1 + C_2 B_2 + C_3 B_3$ does not change the observable projection onto plane of $I_{S(\beta_1,\beta_2,\beta_3)} = \frac{\beta_1 B_1 + \beta_2 B_2 + \beta_3 B_3}{\sqrt{\beta_1^2+\beta_2^2+\beta_3^2}}$ and just flips the observable projection onto plane $I_{S(\beta_2,-\beta_1,-\alpha)} = \frac{\beta_2 B_1 - \beta_1 B_2 - \alpha B_3}{\sqrt{\alpha^2+\beta_1^2+\beta_2^2}}$.

4. Evolution of G-Qubit States

Measurement of observable C by a state $e^{I_S\varphi}$ is defined as $e^{-I_S\varphi}Ce^{I_S\varphi}$. Evolution of a state is its movement on surface of $\mathbb{S}^3$.

Consider necessary formalism.

Multiplication of two geometric algebra exponents reads, see Sec.1.2 of [5]:

$$\begin{aligned} e^{I_{S_1}\alpha}e^{I_{S_2}\beta} &= \left(\cos\alpha + I_{S_1}\sin\alpha\right)\left(\cos\beta + I_{S_2}\sin\beta\right) \\ &= \cos\alpha\cos\beta + I_{S_1}\sin\alpha\cos\beta + I_{S_2}\cos\alpha\sin\beta + I_{S_1}I_{S_2}\sin\alpha\sin\beta \end{aligned}$$

It follows from the formula for bivector multiplication:

$$g_1g_2 = \alpha_1\alpha_2 - \left(s_1\cdot s_2\right)\beta_1\beta_2 + I_{S_1}\alpha_2\beta_1 + I_{S_2}\alpha_1\beta_2 - I_3\left(s_1\times s_2\right)\beta_1\beta_2$$

with vectors to which the unit bivectors I_{S_1} and I_{S_2} are duals: $s_1 = -I_3I_{S_1}$, $s_2 = -I_3I_{S_2}$.

In the current case

$$\alpha_1 = \cos\alpha\,,\quad \alpha_2 = \cos\beta\,,\quad \beta_1 = \sin\alpha\,,\quad \beta_2 = \sin\beta\,,$$

and we get above formula for $e^{I_{S_1}\alpha}e^{I_{S_2}\beta}$.

The product of two exponents is again an exponent, because generally $\left|g_1g_2\right| = \left|g_1\right|\left|g_2\right|$ and $\left|e^{I_{S_1}\alpha}e^{I_{S_2}\beta}\right| = \left|e^{I_{S_1}\alpha}\right|\left|e^{I_{S_2}\beta}\right| = 1$, see Sec.1.3 of [5].

Multiplication of an exponent by another exponent is often called *Clifford translation*. Using the term *translation* follows from the fact that Clifford translation does not change distances between the exponents it acts upon when we identify exponents as points on unit sphere $\mathbb{S}^3$:

$$\cos\alpha + I_S\sin\alpha = \cos\alpha + b_1\sin\alpha B_1 + b_2\sin\alpha B_2 + b_3\sin\alpha B_3$$

$$\Leftrightarrow \left\{\cos\alpha, b_1\sin\alpha, b_2\sin\alpha, b_3\sin\alpha\right\}$$

$$\left(\cos\alpha\right)^2 + \left(b_1\sin\alpha\right)^2 + \left(b_2\sin\alpha\right)^2 + \left(b_3\sin\alpha\right)^2 = 1$$

This result follows again from $\left|g_1g_2\right| = \left|g_1\right|\left|g_2\right|$:

$$\left|e^{I_S\alpha}\left(g_1 - g_2\right)\right| = \left|e^{I_S\alpha}\right|\left|g_1 - g_2\right| = \left|g_1 - g_2\right|$$

Assume the angle α in Clifford translation is a variable one. Then in the case $I_{S_1} = const$:

$$\frac{\partial}{\partial\alpha}e^{I_{S_1}\alpha} = I_{S_1}e^{I_{S_1}\alpha}$$

If I_{S_1} is dual to some unit vector *H*, $I_{S_1} = -I_3H$ (this is the case of the matrix Hamiltonian map to G_3^+, see [3]), then $e^{I_{S_1}\alpha} = e^{-I_3H\alpha} \equiv \psi\left(H,\alpha\right)$ and

$$\frac{\partial}{\partial\alpha}\psi\left(H,\alpha\right) = -I_3H\psi\left(H,\alpha\right)$$

that is obviously Geometric Algebra generalization of the Schrodinger equation.

If vector H varies in time we get, assuming for example $\alpha \equiv t$:

$$\frac{\partial}{\partial t}\psi\left(H(t),t\right) = I_3\left(-H(t) - t\frac{\partial}{\partial t}H(t)\right)\psi\left(H(t),t\right)$$

with, generally, $\psi\left(H(t),t\right)=\mathrm{e}^{-I_3\left(\frac{H(t)}{|H(t)|}\right)|H(t)|t}$.

Assume again constant H and its unit length, $|H|=1$. We see that displacement with $\alpha=\Delta t$ along big circle, intersection of the unit sphere $\mathbb{S}^3$ by plane $-I_3H$, rotates $\psi(H,t)$ lying on $\mathbb{S}^3$ by angle Δt in that plane.

Let us take two planes orthogonal to the plane of $-I_3H$ and comprising right-hand screw with it: $-I_3H_1$ and $-I_3H_2$. Right-handedness means:

$$(-I_3H)(-I_3H_1)=I_3H_2,$$

$$(-I_3H)(-I_3H_2)=-I_3H_1 \quad \text{and}$$

$$(-I_3H_1)(-I_3H_2)=-I_3H$$

(See the earlier definition of the right-hand oriented triple of basis bivectors.) Then the three above formulas mean that the planes $-I_3H_1$ and $-I_3H_2$ rotate synchronically with $-I_3H$, correspondingly in planes $-I_3H_2$ and $-I_3H_1$. Thus, the triple of planes rotates as solid while moving along big circle on $\mathbb{S}^3$.

5. Model of Hydrogen Atom

Let the state has the Hamiltonian type of the form:

$$\psi\left(H(t),t\right)=\mathrm{e}^{-I_3\left(\frac{H(t)}{|H(t)|}\right)|H(t)|t} \tag{5.1}$$

where $H(t)$ is vector in three dimensions. An observable it will act upon is something of a torsion kind, $|r|\mathrm{e}^{I_S\omega t}$. Thus, at instant of time t we have the following result of action of state (5.1):

$$\mathrm{e}^{I_3\left(\frac{H(t)}{|H(t)|}\right)|H(t)|t}|r|\mathrm{e}^{I_S\omega t}\mathrm{e}^{-I_3\left(\frac{H(t)}{|H(t)|}\right)|H(t)|t} \tag{5.2}$$

The Hamiltonian type wave function (5.1) bears its origin from proton, while the observable $|r|\mathrm{e}^{I_S\omega t}$ represents electron.

The geometric algebra existence of the hydrogen atom can only follow from stable sequence of measurement results (5.2) with appropriate combination(s) of $H(t)$ and ω.

Let

$$H(t)=-h_1(t)I_3B_1-h_2(t)I_3B_2-h_3(t)I_3B_3$$

Then $|H(t)|=\sqrt{h_1^2(t)+h_2^2(t)+h_3^2(t)}$, bivector part of (5.1) is $\frac{\sin\left(|H(t)|t\right)}{|H(t)|}\left(h_1(t)B_1+h_2(t)B_2+h_3(t)B_3\right)$ and the scalar part of the wave function (5.1) is $\cos\left(|H(t)|t\right)$.

If initial bivector plane of observable is $c_1B_1+c_2B_2+c_3B_3$, $c_1^2+c_2^2+c_3^2=1$, scalar part then is $|r|\cos\omega t$, thus

$|r|\mathrm{e}^{I_S\omega t}=|r|\cos\omega t+|r|\sin\omega t\left(c_1(t)B_1+c_2(t)B_2+c_3(t)B_3\right)$.

Let us denote the plane $-I_3\left(\frac{H(t)}{|H(t)|}\right) \equiv I_H$. Then the sequence of transformations (5.2) reads:

$$\mathrm{e}^{-I_H|H|\Delta t}\left(\cdots\left(\mathrm{e}^{-I_H|H|\Delta t}\left(\mathrm{e}^{-I_H|H|t}|r|\mathrm{e}^{I_S\omega t}\mathrm{e}^{I_H|H|t}\right)\mathrm{e}^{I_H|H|\Delta t}\right)\cdots\right)\mathrm{e}^{I_H|H|\Delta t}$$

If $I_S = I_H$ and assuming that $H(t)$ does not depend on time. we get:

$$|r|\mathrm{e}^{-I_H|H|t}\mathrm{e}^{I_H\omega t}\mathrm{e}^{I_H|H|t}$$

Angular velocity ω should be synchronized with Hamiltonian rotation by $2|H|$, though it can be integer times greater than $2|H|$.

Now assume that $I_S \neq I_H$. Thus, the result of (5.2) is:

$$|r|\mathrm{e}^{-I_H|H|t}\mathrm{e}^{I_S\omega t}\mathrm{e}^{I_H|H|t}$$

The vector of length $|r|$ rotates in plane I_S with angular velocity ω while element $|r|\mathrm{e}^{I_S\omega t}$ rotates in plane I_H. Again, for stability, angular velocity ω should be integer times greater than $2|H|$.

Take the general formula (3.1) and substitute $C_0 = |r|\cos\omega t$, $C_1 = |r|c_1\sin\omega t$, $C_2 = |r|c_2\sin\omega t$, $C_3 = |r|c_3\sin\omega t$, where c_i are components of I_S in the basis $\{B_1, B_2, B_3\}$, and $\alpha = \cos(|H|t)$, $\beta_i = h_i\sin(|H|t)$, h_i are components of I_H in the basis $\{B_1, B_2, B_3\}$: $I_H = h_1B_1 + h_2B_2 + h_3B_3$. The result of measurement, after multiple transformations, reads:

$$\begin{aligned}
&\frac{|r|\sin 2|H|t}{2|H|^2}\Big\{\Big[c_1\left(\left(1+\cos 2|H|t\right)|H|^2+\left(1-\cos 2|H|t\right)\left(2h_1^2-|H|^2\right)\right)\\
&+2c_2\left(\left(1-\cos 2|H|t\right)h_1h_2-|H|h_3\sin 2|H|t\right)+2c_3\left(\left(1-\cos 2|H|t\right)h_1h_3\right.\\
&\left.+|H|\sin 2|H|th_2\right)\Big]B_1+\Big[2c_1\left(\left(1-\cos 2|H|t\right)h_1h_2-|H|h_3\sin 2|H|t\right)\\
&+c_2\left(\left(1+\cos 2|H|t\right)|H|^2+\left(1-\cos 2|H|t\right)\left(2h_2^2-|H|^2\right)\right)\\
&+2c_3\left(\left(1-\cos 2|H|t\right)h_2h_3-|H|\sin 2|H|th_1\right)\Big]B_2\\
&+\Big[2c_1\left(\left(1-\cos 2|H|t\right)h_1h_3-|H|\sin 2|H|th_2\right)+2c_2\left(\left(1-\cos 2|H|t\right)h_2h_3\right.\\
&\left.-|H|\sin 2|H|th_1\right)+c_3\left(\left(1+\cos 2|H|t\right)|H|^2+\left(1-\cos 2|H|t\right)\left(2h_3^2-|H|^2\right)\right)\Big]B_3\Big\}
\end{aligned} \tag{5.3}$$

Formula (5.3) gives stable rotation of observable $|r|\cos\omega t + |r|\sin\omega t\left(c_1(t)B_1 + c_2(t)B_2 + c_3(t)B_3\right)$ (electron) due to action of the state $\psi(H(t), t) = \mathrm{e}^{-I_3\left(\frac{H(t)}{|H(t)|}\right)|H(t)|t}$ (proton.)

6. Conclusion

It was demonstrated that the geometric algebra formalism along with generalization of complex numbers and subsequent lift of the two-dimensional Hilbert space valued qubits to geometrically feasible elements of even subalgebra of geometric algebra in three dimensions allows, particularly, to explain what actually means senseless “find system in state”. The approach also supports elimination of primitive Bohr’s planetary model of the hydrogen atom.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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