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# Instantaneous Spreading of the g-Qubit Fields

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## Abstract

The Geometric Algebra formalism opens the door to developing a theory deeper than conventional quantum mechanics. Generalizations, stemming from implementation of complex numbers as geometrically feasible objects in three dimensions, unambiguous definition of states, observables, measurements, Maxwell equations solution in that terms, bring into reality a kind of physical fields, states in the suggested theory, spreading through the whole three-dimensional space and values of the time parameter. The fields can be modified instantly in all points of space and time values, thus eliminating the concept of cause and effect and perceiving of one-directional time.

## Keywords

Quantum States, Clifford Translations, Geometric Algebra

## 1. Introduction. Geometric Algebra in Three Dimensions

The living space of objects in the suggested theory is geometric (often called Clifford) algebra in three dimensions, $G_3$.

General element of the algebra is sum of scalar, vector, bivector and pseudoscalar:

$$M_3 = (M)_S + (M)_V + (M)_B + (M)_P$$

The geometric algebra sum of geometrical objects of non-similar types should be thought about as the objects "putting together in one bag", not as "pouring some amounts of liquids in one glass, shaken not stirred!".

The $(M)_V$ elements are vectors $a, b, c, \cdots$ of the three-dimensional Euclidean space $E_3$.

Bivectors, denoted by capitals $A, B, C, \cdots$, are identified by planes they belong to, the value of bivector area (value of bivector) and its orientation. The shape of the bivector area boundary does not matter if the inside area remains the same.

Trivectors, or pseudoscalars, are defined by value, which is volume, and one of two possible orientations—right screw handedness or left screw handedness. Actually, trivector is class of equivalence of volumes of the same value and one of two possible volume orientations. A trivector class particularly contains parallelepipeds with vector edges. If the edges compose right-hand screw triple, we will say that trivector has sign (equivalent to orientation) *plus*. If the edges comprise left-hand screw triple, the trivector has sign *minus*. Parallelepipeds can be deformed in any (continuous) way not changing the volume (as a piece of clay in sculpture hands)—trivector remains the same. Unit value pseudpscalar is denoted $I_3$.

If *a* is vector, then $I_3 a = aI_3$ is bivector the contour of which bounds area equal to the length of *a*. The contour plane is orthogonal to vector *a*. The orientation of the bivector, the direction of movement along its contour, together with the direction of *a* should match the orientation of $I_3$.

If $A$ is a bivector then $I_3 A = AI_3$ is vector of length equal to the area of bivector $A$. This vector is orthogonal to the plane of $A$. The direction of vector $I_3 A$ should make, together with the $A$ orientation, the screw *opposite* to $I_3$.

Vector *a* received from bivector $A$ as $I_3 A$ is called dual to $A$. If bivector $A$ is $I_3 a$ then $A$ is called dual to *a*.

Using the duality between different types of elements of $G_3$ an arbitrary element $M_3 \in G_3$ can be written as:

$$M_3 = \alpha + a + I_3 b + I_3 \beta$$

where $\alpha$ and $\beta$ are scalars, and *a*, *b*—vectors of $E_3$.

Algebra $G_3$ has two types of conjugation, order conjugation:

$$\bar{M}_3 = (M)_S + (M)_V - (M)_B - (M)_P$$

and conjugation of direction:

$$\tilde{M}_3 = (M)_S - (M)_V + (M)_B - (M)_P$$

Equivalently:

$$\bar{M}_3 = \alpha + a - I_3 b - I_3 \beta$$

$$\tilde{M}_3 = \alpha - a + I_3 b - I_3 \beta$$

Linear space $G_3$ can be supplied with norm:

$$\|M_3\|^2 = \left(M_3 \bar{M}_3\right)_S = \alpha^2 + |a|^2 + |b|^2 + \beta^2$$

The elements of $G_3$ satisfying $M_3 = \tilde{M}_3$, that's of the form $M_3 = \alpha + I_S \beta$, where $I_S$ is some unit bivector arbitrary placed in three dimensional space, bear the name of even elements of algebra $G_3$. The even elements make their own subalgebra because operations of algebraic addition and multiplication of even elements return even elements. The subalgebra of even elements will be denoted by $G_3^+$.

## 2. States, Observables, Measurements

Unambiguous definition of states and observables, does not matter are we in "classical" or "quantum" frame, should follow the general paradigm [1] [2] [3]

- Measurement of observable $O(\mu)$ in state[1] $S(\lambda)$ is a map:

$$\big(S(\lambda), O(\mu)\big) \to O(\nu),$$

where $O(\mu)$ is an element of the set of observables. $S(\lambda)$ is element of another set, set of states, though both sets can be formally equivalent.

- The result (value) of a measurement of observable $O(\mu)$ by the state $S(\lambda)$ is a map sequence

$$\big(S(\lambda), O(\mu)\big) \to O(\nu) \to V(B),$$

where $V$ is a set of (Boolean) algebra subsets identifying possible results of measurements.

The importance of the above definitions becomes obvious even from trivial examples.

Let's take a point moving along straight line. The definitions are pictured as (**Figure 1**).

In this example it does not formally matter do we consider evolution of "state" or of "measurement of observable by the state" or of "the result of measurement" because they differ only by an additive constant or the map of one-dimensional vector to its length. In the conventional quantum mechanics similar formal identifications are commonly followed without justification.

The above one-dimensional situation radically changes if the process entities become belonging to a plane, that's dimensionality of physical process increases, though we continue watching results in one dimensional projection (**Figure 2**).

In a not deterministic evolution the randomness of observed values is due to the fact that their probabilities are associated with partition of the space of states (**Figure 3**). Each partition element is fiber (level set)[2] of each of the observable value under the action of the state on observable. Probabilities are (relative) measures of those fibers.

The option to expand, to lift the space where physical processes are considered, may have critical consequence to a theory. A kind of expanding is the core of the suggested formulation.

## 3. Lift of Qubits to g-Qubits

The very first critical thing for the whole approach is to generalize algebraically formal two-dimensional complex number vectors to geometrically clear, unambiguous objects—elements of even subalgebra $G_3^+$ of geometric algebra over the three-dimensional space. Such objects are identified by an arbitrary oriented plane in three dimensions and angle of rotation in that plane. I will call such objects *g-qubits*, if they have unit value, to distinguish them from qubits as

[1]Correctly would be to say "by a state". State is operator acting on observable.

[2]Recall that fiber of a point $y$ in $Y$ under a function $f: X \to Y$ is the inverse image of $\{y\}$ under $f$: $f^{-1}(\{y\}) = \{x \in X : f(x) = y\}$.

complex valued two-dimensional unit value vectors, see Figure 4. Thus, g-qubit is element of the $G_3^+$, geometrical algebra sum of a scalar and bivector, $\alpha + I_S\beta$, with $\alpha^2+\beta^2=1$, and $I_S$ is unit value, oriented plane in 3D. The plane of g-qubit does not generally coincide with the planes of observables it is applied to. Observables are also object of $G_3^+$.

Measurement is by definition the result of action of operator, state g-qubit ($\alpha + I_S\beta$), on an observable $C \in G_3^+$:

$$(\alpha - I_S\beta)C(\alpha + I_S\beta) = \widetilde{(\alpha + I_S\beta)}\,C(\alpha + I_S\beta)$$

It can be conveniently written in exponential form:

$$\mathrm{e}^{-I_S\varphi}C\mathrm{e}^{I_S\varphi}, \text{ where } \varphi = \cos^{-1}\alpha$$

The g-qubit on the right side of Figure 4 has the right-hand screw orientation.

Take arbitrary qubit $\begin{pmatrix} x_1+iy_1 \\ x_2+iy_2 \end{pmatrix}$, $\|x_1+iy_1\|^2+\|x_2+iy_2\|^2=1$, unit value element of $C^2$. Its lift to $G_3^+$ is defined as:

$$\begin{pmatrix} x_1+iy_1 \\ x_2+iy_2 \end{pmatrix} \Rightarrow x_1+y_1B_1+y_2B_2+x_2B_3 = x_1+y_1B_1+y_2B_1B_3+x_2B_3$$
$$= x_1+y_1B_1+\left(x_2+y_2B_1\right)B_3$$

where $\{B_1,B_2,B_3\}$ is an arbitrary triple of unit value bivectors in three dimensions satisfying, with not critical assumption of right-hand screw orientation $B_1B_2B_3=1$, the multiplication rules, see Figure 5:

$$B_1B_2=-B_3,\quad B_1B_3=B_2,\quad B_2B_3=-B_1$$

The lift uses $\{B_1,B_2,B_3\}$ reference frame of unit value bivectors. The frame, as a solid, can be arbitrary rotated in three dimensions. In that sense we have principal fiber bundle $G_3^+ \to C^2$ with the standard fiber as group of rotations which is also effectively identified by elements of $G_3^+$.

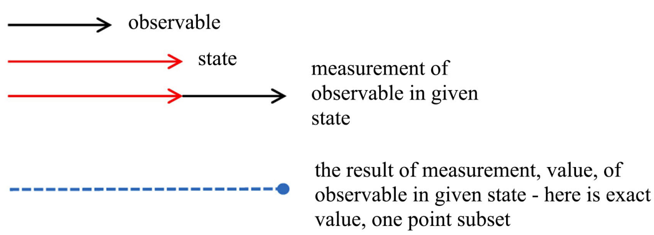


Figure 1. A state acts on observable in one-dimensional movement.

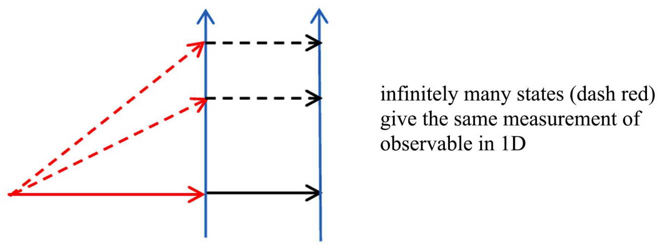


Figure 2. State acts in two dimensions though the result is available just as projection.

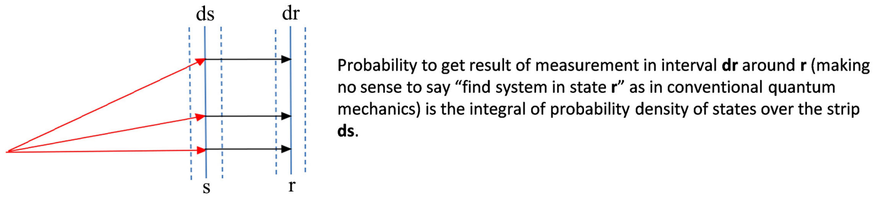


**Figure 3.** Probabilistic distribution of states results in probabilistic measurements.

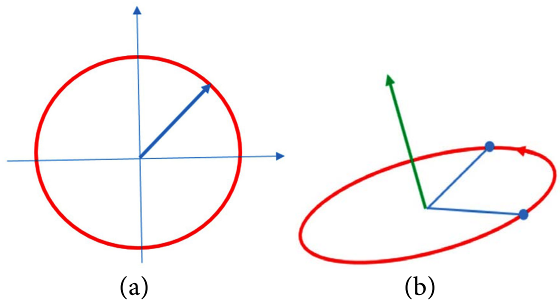


**Figure 4.** (a) Conventional quantum mechanics qubit is a couple of such complex numbers, vectors rotated by an angle in unspecified plane; (b) g-Qubit is a unit value area in oriented plane in three dimensions together with angle of rotation in that plane.

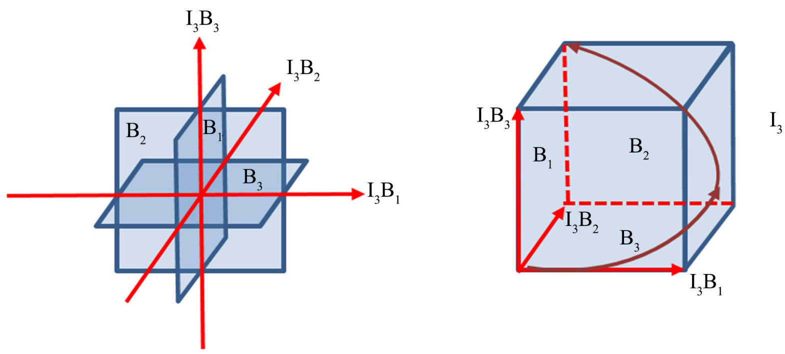


**Figure 5.** Basis of bivectors and unit value pseudoscalar.

## 4. Maxwell Equation in Geometric Algebra

Let's show how the system of the electromagnetic Maxwell equations is formulated as one equation in geometric algebra terms [4].

Take geometric algebra element of the form: $F = e + I_3 h$. The electromagnetic field $F$ is created by some given distribution of charges and currents, also written as geometric algebra multivector: $J \equiv \rho - j$. Apply operator $\partial_t + \nabla$, where $\nabla = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}$ [3] and multiplication is the geometrical algebra one, to the $F$. The result is:

$$(\partial_t + \nabla)F = \underbrace{\nabla \cdot e}_{\text{scalar}} + \underbrace{\partial_t e + I_3(\nabla \wedge h)}_{\text{vector}} + \underbrace{\nabla \wedge e + I_3 \partial_t h}_{\text{bivector}} + \underbrace{I_3(\nabla \cdot h)}_{\text{pseudoscalar}}$$

Comparing component-wise $(\partial_t + \nabla)F$ and $J$ we get:

[3] For any vector we write $\hat{a} = a/|a|$

$$\begin{cases} \nabla \cdot e \equiv dive = \rho \\ \partial_t e + I_3 (\nabla \wedge h) \equiv \partial_t e - roth = -j \\ \nabla \wedge e + I_3 \partial_t h \equiv I_3 rote + I_3 \partial_t h = 0 \\ I_3 (\nabla \cdot h) \equiv I_3 (divh) = 0 \end{cases}$$

Thus, we have usual system of Maxwell equations:

$$\begin{cases} dive = \rho \\ \partial_t e - roth = -j \\ \partial_t h + rote = 0 \\ divh = 0 \end{cases}$$

equivalent to one equation $(\partial_t + \nabla) F = J$.

Without charges and currents the equation becomes

$$(\partial_t + \nabla) F = 0 \tag{4.1}$$

The circular polarized electromagnetic waves are the only type of waves following from the solution of Maxwell equations in free space done in geometric algebra terms.

Indeed, let's take the electromagnetic field in the form:

$$F = F_0 \exp\left[ I_S (\omega t - k \cdot r) \right] \tag{4.2}$$

requiring that it satisfies (4.1).

Element $F_0$ in (4.2) is a constant element of geometric algebra $G_3$ and $I_S$ is unit value bivector of a plane $S$ in three dimensions, generalization of the imaginary unit [1] [5]. The exponent in (4.2) is unit value element of $G_3^+$ [1]:

$$\mathrm{e}^{I_S \varphi} = \cos \varphi + I_S \sin \varphi, \quad \varphi = \omega t - k \cdot r$$

Solution of (4.1) should be sum of a vector (electric field *e*) and bivector (magnetic field $I_3 h$):

$$F = e + I_3 h$$

with some initial conditions:

$$e + I_3 h\big|_{t=0,\vec{r}=0} = F_0 = e\big|_{t=0,\vec{r}=0} + I_3 h\big|_{t=0,\vec{r}=0} = e_0 + I_3 h_0$$

Substitution of (4.2) into the Maxwell's (4.1) will show us what the solution looks like.

The derivative by time gives

$$\frac{\partial}{\partial t} F = F_0 \mathrm{e}^{I_S \varphi} I_S \frac{\partial}{\partial t} (\omega t - k \cdot r) = F_0 \mathrm{e}^{I_S \varphi} I_S \omega = F I_S \omega$$

The geometric algebra product $\nabla F$ is:

$$\nabla F = F_0 I_S \mathrm{e}^{I_S \varphi} \nabla (\omega t - k \cdot r) = -F_0 \mathrm{e}^{I_S \varphi} I_S k = -F I_S k$$

or

$$\nabla F = F_0 \mathrm{e}^{I_S \varphi} \nabla (\omega t - k \cdot r) I_S = -F_0 \mathrm{e}^{I_S \varphi} k I_S = -F k I_S,$$

depending on do we write $I_S (\omega t - k \cdot r)$ or $(\omega t - k \cdot r) I_S$. The result should be the same because $\omega t - k\cdot r$ is a scalar.

Commutativity $I_S k = kI_S$ is valid only if $k \times I_3 I_S = 0$. The following agreement takes place between orientation of $I_3$, orientation of $I_S$ and direction of vector $k$ [1]. The vector $I_3 I_S = I_S I_3$ is orthogonal to the plane of $I_S$ and its direction is defined by orientations of $I_3$ and $I_S$. Rotation of right/left hand screw defined by orientation of $I_S$ gives movement of right/left hand screw. This is the direction of the vector $I_3 I_S = I_S I_3$. That means that the matching between $\hat{k}$ and $I_S$ should be $\hat{k} = \pm I_3 I_S \Rightarrow \hat{k} I_S = \mp I_3$.

Assume first that orientation is $I_3 = \hat{k} I_S$. Then Maxwell equation becomes:

$$F\left(I_S \omega - I_3 |k|\right) = F\left(\omega I_S - |k|\hat{k} I_S\right) = 0$$

$$\text{or} \quad (e + I_3 h)\omega = (e + I_3 h)k$$

Left hand side of equation is sum of vector and bivector, while right hand side is scalar $e\cdot k$ plus bivector $e \wedge k$, plus pseudoscalar $I_3(h\cdot k)$, plus vector $I_3(h \wedge k)$. It follows that both $e$ and $h$ lie on the plane of $I_S$ and then:

$$\omega e = I_3 hk, \quad \omega I_3 h = ek \quad \rightarrow \frac{\omega^2}{|k|^2} I_3 hk = \omega e$$

Thus, $\omega = |k|$ and we get equation $I_3 h\hat{k} = e$ from which particularly follows $|e|^2 = |h|^2$ and $\hat{e}\hat{k}\hat{h} = I_3$.

The result for the case $I_3 = \hat{k} I_S$ is that the solution of (4.1) is

$$F = (e_0 + I_3 h_0)\exp\left[I_S(\omega t - k\cdot r)\right]$$

where $e_0$ and $h_0$ are arbitrary mutually orthogonal vectors of equal length, lying on the plane *S*. Vector $k$ should be normal to that plane, $\hat{k} = -I_3 I_S$ and $|k| = \omega$.

In the above result the sense of the $I_S$ orientation and the direction of $k$ were assumed to agree with $I_3 = \hat{k} I_S$. Opposite orientation, $-I_3 = \hat{k} I_S$, that's $k$ and $I_S$ compose left hand screw and $\hat{k} = I_3 I_S$, will give solution $F = (e_0 + I_3 h_0)\exp\left[I_S(\omega t - k\cdot r)\right]$ with $\hat{e}\hat{h}\hat{k} = I_3$.

Summary:

For a plane *S* in three dimensions Maxwell Equation (4.1) has two solutions

- $F_+ = (e_0 + I_3 h_0)\exp\left[I_S(\omega t - k_+\cdot r)\right]$, with $\hat{k}_+ = I_3 I_S$, $\hat{e}\hat{h}\hat{k}_+ = I_3$, and the triple $\left\{\hat{e}, \hat{h}, \hat{k}_+\right\}$ is right hand screw oriented, that's rotation of $\hat{e}$ to $\hat{h}$ by π/2 gives movement of *right hand screw* in the direction of $k_+ = |k| I_3 I_S$.
- $F_- = (e_0 + I_3 h_0)\exp\left[I_S(\omega t - k_-\cdot r)\right]$, with $\hat{k}_- = -I_3 I_S$, $\hat{e}\hat{h}\hat{k}_- = -I_3$, and the triple $\left\{\hat{e}, \hat{h}, \hat{k}_-\right\}$ is left hand screw oriented, that's rotation of $\hat{e}$ to $\hat{h}$ by π/2 gives movement of *left hand screw* in the direction of $k_- = -|k| I_3 I_S$ or, equivalently, movement of *right hand screw* in the opposite direction, $-k_-$.
- $e_0$ and $h_0$, initial values of *e* and *h*, are arbitrary mutually orthogonal vectors of equal length, lying on the plane *S*. Vectors $k_\pm = \pm|k_\pm| I_3 I_S$ are normal to that plane. The length of the "wave vectors" $|k_\pm|$ is equal to angular frequency $\omega$.

Maxwell Equation (4.1) is a linear one. Then any linear combination of $F_+$

and $F_-$ saving the structure of (4.2) will also be a solution.

Let's write:

$$\begin{cases} F_+ = (e_0 + I_3 h_0)\exp\left[I_S\omega\left(t-(I_3 I_S)\cdot r\right)\right] = (e_0 + I_3 h_0)\exp\left[I_S\omega t\right]\exp\left[-I_S\left[(I_3 I_S)\cdot r\right]\right] \\ F_- = (e_0 + I_3 h_0)\exp\left[I_S\omega\left(t+(I_3 I_S)\cdot r\right)\right] = (e_0 + I_3 h_0)\exp\left[I_S\omega t\right]\exp\left[I_S\left[(I_3 I_S)\cdot r\right]\right] \end{cases} \tag{4.3}$$

Then for arbitrary (real[4]) scalars $\lambda$ and $\mu$:

$$\lambda F_+ + \mu F_- = (e_0 + I_3 h_0)\mathrm{e}^{I_S\omega t}\left(\lambda \mathrm{e}^{-I_S\left[(I_3 I_S)\cdot r\right]} + \mu \mathrm{e}^{I_S\left[(I_3 I_S)\cdot r\right]}\right) \tag{4.4}$$

is solution of (4.1). The item in the second parenthesis is weighted linear combination of two states with the same phase in the same plane but opposite sense of orientation. The states are strictly coupled, entangled if you prefer, because bivector plane should be the same for both, does not matter what happens with that plane.

One another option of linear combination saving the structure of (4.2) is:

$$\begin{aligned} &(\lambda_1 + I_3\mu_1)(e_0 + I_3 h_0)\exp\left[I_S\omega\left(t-(I_3 I_S)\cdot r\right)\right] \\ &+(\lambda_2 + I_3\mu_2)(e_0 + I_3 h_0)\exp\left[I_S\omega\left(t+(I_3 I_S)\cdot r\right)\right] \\ &=\left[\lambda_1 e_0 - \mu_1 h_0 + I_3(\mu_1 e_0 + \lambda_1 h_0)\right]\exp\left[I_S\omega\left(t-(I_3 I_S)\cdot r\right)\right] \\ &\quad+\left[\lambda_2 e_0 - \mu_2 h_0 + I_3(\mu_2 e_0 + \lambda_2 h_0)\right]\exp\left[I_S\omega\left(t+(I_3 I_S)\cdot r\right)\right] \end{aligned}$$

which is just rotation, along with possible change of length, of electric and magnetic initial vectors in their plane.

Arbitrary linear combination (4.4) can be rewritten as:

$$\lambda \mathrm{e}^{I^+_{Plane}\varphi^+} + \mu \mathrm{e}^{I^-_{Plane}\varphi^-} \tag{4.5}$$

where

$$\varphi^{\pm} = \cos^{-1}\left(\frac{1}{\sqrt{2}}\cos\omega\left(t \mp \left[(I_3 I_S)\cdot r\right]\right)\right),$$

$$\begin{aligned} I^{\pm}_{Plane} = I_S \frac{\sin\omega\left(t\mp\left[(I_3 I_S)\cdot r\right]\right)}{\sqrt{1+\sin^2\omega\left(t\mp\left[(I_3 I_S)\cdot r\right]\right)}} + I_{B_0}\frac{\cos\omega\left(t\mp\left[(I_3 I_S)\cdot r\right]\right)}{\sqrt{1+\sin^2\omega\left(t\mp\left[(I_3 I_S)\cdot r\right]\right)}} \\ + I_{E_0}\frac{\sin\omega\left(t\mp\left[(I_3 I_S)\cdot r\right]\right)}{\sqrt{1+\sin^2\omega\left(t\mp\left[(I_3 I_S)\cdot r\right]\right)}} \end{aligned}$$

The triple of unit value basis orthonormal bivectors $\{I_S, I_{B_0}, I_{E_0}\}$ is comprised of the $I_S$ bivector, dual to the propagation direction vector; $I_{B_0}$ is dual to initial vector of magnetic field; $I_{E_0}$ is dual to initial vector of electric field. The expression (4.5) is linear combination of two geometric algebra states, g-qubits.

## 5. Clifford Translations of States (4.5)

For the further considerations we need the notion of *Clifford translations* acting

[4]Remember, in the current theory scalars are real ones. "Complex" scalars have no sense.

on states. Clifford translation by $\gamma$ (scalar) in an arbitrary plane $B_C$ acts, by definition, on an arbitrary state $e^{I_B \varphi}$ as:

$$e^{I_B \varphi} \to e^{I_{B_C} \gamma} e^{I_B \varphi}.$$

Linear combination of the two equally weighted basic solutions of the Maxwell equation $F_+$ and $F_-$, $\lambda F_+ + \mu F_-$ with $\lambda = \mu = 1$ reads:

$$\lambda F_+ + \mu F_- \big|_{\lambda=\mu=1}$$
$$= 2\cos\omega\left[\left(I_3 I_S\right)\cdot r\right]\left(\frac{1}{\sqrt{2}}\cos\omega t + I_S \frac{1}{\sqrt{2}}\sin\omega t + I_{B_0}\frac{1}{\sqrt{2}}\cos\omega t + I_{E_0}\frac{1}{\sqrt{2}}\sin\omega t\right) \quad (4.6)$$

where $\cos\varphi = \frac{1}{\sqrt{2}}\cos\omega t$ and $\sin\varphi = \frac{1}{\sqrt{2}}\sqrt{1+\left(\sin\omega t\right)^2}$. It can be written in standard exponential form $\cos\varphi + \sin\varphi I_B = e^{I_B \varphi}$.[5]

I will call such g-qubits ***spreons*** because they spread over the whole three-dimensional space for all values of time and instantly change under Clifford translations over the whole three-dimensional space for all values of time, along with the results of measurement of any observable.

*Remark* 2.1:

If Clifford translation of a state $e^{I_S(t)\varphi(t)}$ is associated with a Hamiltonian, that's the translation is $e^{-I_3 \frac{H(t_0)}{|H(t_0)|}|H(t_0)|\Delta t} e^{I_S(t_0)\varphi(t_0)}$, where Hamiltonian $H(t) = \alpha + I_3\left(\beta B_1 + \gamma B_2 + \delta B_3\right)$ is lift of a Hermitian matrix $\begin{pmatrix} \alpha+\beta & \gamma - i\delta \\ \gamma + i\delta & \alpha - \beta \end{pmatrix}$, and $I_3 \frac{H(t_0)}{\left|H(t_0)\right|} \equiv I_H(t_0)$ is generalization of imaginary unit in the current theory, then:

$$e^{I_S(t_0+\Delta t)\varphi(t_0+\Delta t)} = e^{-I_H(t_0)|H(t_0)|\Delta t} e^{I_S(t_0)\varphi(t_0)}$$

and

$$\lim_{\Delta t\to 0}\frac{\Delta e^{I_S(t_0)\varphi(t_0)}}{\Delta t} = \lim_{\Delta t\to 0}\frac{e^{I_S(t_0+\Delta t)\varphi(t_0+\Delta t)} - e^{I_S(t_0)\varphi(t_0)}}{\Delta t}$$
$$= \lim_{\Delta t\to 0}\frac{\left(1 - I_H(t_0)\left|H(t_0)\right|\Delta t\right)e^{I_S(t_0)\varphi(t_0)} - e^{I_S(t_0)\varphi(t_0)}}{\Delta t}$$
$$= -I_H(t_0)\left|H(t_0)\right| e^{I_S(t_0)\varphi(t_0)}$$

that immediately gives the Schrodinger equation for the state $e^{I_S(t)\varphi(t)}$. That means that Schrodinger equation governs evolution of operators, states, which act on observables.

*End of the Remark* 2.1.

Arbitrary Clifford translation $e^{I_{B_C}\gamma} = \cos\gamma + \sin\gamma\left(\gamma_1 I_S + \gamma_2 I_{B_0} + \gamma_3 I_{E_0}\right)$ acting

[5]Good to remember that the two basic solutions $F_+$ and $F_-$ differ only by the sign of $I_3 I_S$, which is caused by orientation of $I_S$ that in its turn defines if the triple $\left\{\hat{E}, \hat{H}, \pm I_3 I_S\right\}$ is right-hand screw or left-hand screw oriented.

on spreons (4.6) gives:

$$
\begin{aligned}
&2\cos\omega\left[\left(I_3 I_S\right)\cdot r\right] \\
&\cdot\left[\frac{1}{\sqrt{2}}\left(\cos\gamma\cos\omega t-\gamma_1\sin\gamma\sin\omega t-\gamma_2\sin\gamma\cos\omega t-\gamma_3\sin\gamma\sin\omega t\right)\right. \\
&+\frac{1}{\sqrt{2}}\left(\cos\gamma\sin\omega t+\gamma_1\sin\gamma\cos\omega t-\gamma_2\sin\gamma\sin\omega t+\gamma_3\sin\gamma\cos\omega t\right)I_S \\
&+\frac{1}{\sqrt{2}}\left(\cos\gamma\cos\omega t+\gamma_1\sin\gamma\sin\omega t+\gamma_2\sin\gamma\cos\omega t-\gamma_3\sin\gamma\sin\omega t\right)I_{B_0} \\
&\left.+\frac{1}{\sqrt{2}}\left(\cos\gamma\sin\omega t-\gamma_1\sin\gamma\cos\omega t+\gamma_2\sin\gamma\sin\omega t+\gamma_3\sin\gamma\cos\omega t\right)I_{E_0}\right]
\end{aligned} \tag{4.7}
$$

This result is defined for all values of $t$ and $r$, in other words the result of Clifford translation instantly spreads through the whole three-dimensions for all values of time.

Measurement of any observable $C_0+C_1B_1+C_2B_2+C_3B_3$ (actually Hopf fibration) by a state $\alpha+\beta_1B_1+\beta_2B_2+\beta_3B_3$ in the current formalism:

$$
\begin{aligned}
&C_0+C_1B_1+C_2B_2+C_3B_3\xrightarrow{\alpha+\beta_1B_1+\beta_2B_2+\beta_3B_3}C_0 \\
&+\left(C_1\left[\left(\alpha^2+\beta_1^2\right)-\left(\beta_2^2+\beta_3^2\right)\right]+2C_2\left(\beta_1\beta_2-\alpha\beta_3\right)+2C_3\left(\alpha\beta_2+\beta_1\beta_3\right)\right)B_1 \\
&+\left(2C_1\left(\alpha\beta_3+\beta_1\beta_2\right)+C_2\left[\left(\alpha^2+\beta_2^2\right)-\left(\beta_1^2+\beta_3^2\right)\right]+2C_3\left(\beta_2\beta_3-\alpha\beta_1\right)\right)B_2 \\
&+\left(2C_1\left(\beta_1\beta_3-\alpha\beta_2\right)+2C_2\left(\alpha\beta_1+\beta_2\beta_3\right)+C_3\left[\left(\alpha^2+\beta_3^2\right)-\left(\beta_1^2+\beta_2^2\right)\right]\right)B_3
\end{aligned}
$$

with:

$$B_1=I_S,\quad B_2=I_{B_0},\quad B_3=I_{E_0},$$

$$
\begin{aligned}
\alpha&=2\cos\omega\left[\left(I_3I_S\right)\cdot r\right]\frac{1}{\sqrt{2}}\left(\cos\gamma\cos\omega t-\gamma_1\sin\gamma\sin\omega t\right. \\
&\quad\left.-\gamma_2\sin\gamma\cos\omega t-\gamma_3\sin\gamma\sin\omega t\right) \\
\beta_1&=2\cos\omega\left[\left(I_3I_S\right)\cdot r\right]\frac{1}{\sqrt{2}}\left(\cos\gamma\sin\omega t+\gamma_1\sin\gamma\cos\omega t\right. \\
&\quad\left.-\gamma_2\sin\gamma\sin\omega t+\gamma_3\sin\gamma\cos\omega t\right) \\
\beta_2&=2\cos\omega\left[\left(I_3I_S\right)\cdot r\right]\frac{1}{\sqrt{2}}\left(\cos\gamma\cos\omega t+\gamma_1\sin\gamma\sin\omega t\right. \\
&\quad\left.+\gamma_2\sin\gamma\cos\omega t-\gamma_3\sin\gamma\sin\omega t\right) \\
\beta_3&=2\cos\omega\left[\left(I_3I_S\right)\cdot r\right]\frac{1}{\sqrt{2}}\left(\cos\gamma\sin\omega t-\gamma_1\sin\gamma\cos\omega t\right. \\
&\quad\left.+\gamma_2\sin\gamma\sin\omega t+\gamma_3\sin\gamma\cos\omega t\right)
\end{aligned}
$$

gives a $G_3^+$ element $O\left(C_0,C_1,C_2,C_3,I_S,I_{B_0},I_{E_0},\gamma,\gamma_1,\gamma_2,\gamma_3,\omega,t,r\right)$ spreading through the three-dimensional space for all values of the time parameter $t$.

The instant of time when the Clifford translation was applied makes no difference for the state (4.7) because the is simultaneously redefined for all values of $t$. The values of measurements $O\left(C_0,C_1,C_2,C_3,I_S,I_{B_0},I_{E_0},\gamma,\gamma_1,\gamma_2,\gamma_3,\omega,t,r\right)$ also get instantly changed for all values of time of measurement, even if the Clifford translation was applied later than the measurement. That is obvious demonstration that the suggested theory allows indefinite event casual order. In that way the very notion of the concept of cause and effect, ordered by time value increasing, disappears.

Since general result of measurement when Clifford translation takes place in an arbitrary plane is pretty complicated, I am only giving the result for the special case $\gamma_1 = 1$ and $\gamma_2 = \gamma_3 = 0$ (Clifford translation acts in plane $I_S$). The result is:

$$
\begin{aligned}
&O\left(C_0, C_1, C_2, C_3, I_S, I_{B_0}, I_{E_0}, \gamma, \gamma_1, \gamma_2, \gamma_3, \omega, t, r\right)_{\gamma_1=1, \gamma_2=\gamma_3=0?} \\
&= 4\cos^2\omega\left[\left(I_3 I_S\right)\cdot r\right]\left[C_0 + \left(C_2 \sin 2\gamma + C_3 \cos 2\gamma\right) I_S \right. \\
&\quad + \left(C_1 \sin 2\omega t + \sin 2\gamma \cos 2\omega t \left(C_2 + C_3\right)\right) I_{B_0} \\
&\quad \left. + \left(-C_1 \cos 2\omega t + \sin 2\gamma \sin 2\omega t \left(C_2 - C_3\right)\right) I_{E_0} \right]
\end{aligned}
$$

The only component of measurement, namely lying in the plane $I_S$, does not change with time[6]. The $I_{B_0}$ and $I_{E_0}$ components do depend on the time of measurement being modified forward and backward in time if Clifford translation is applied. Clifford translation modifies measurement results of the past and the future.

## 6. Conclusions

The seminal ideas: variable and explicitly defined complex plane in three dimensions, the $G_3^+$ states as operators acting on observables, solution of the Maxwell equation(s) in the $G_3$ frame giving $G_3^+$ states, spreons, spreading over the whole three-dimensional space for all values of time, along with the results of measurement of any observable, allow putting forth comprehensive and much more detailed formalism replacing conventional quantum mechanics.

The spreon states, subjected to Clifford translations, change instantly forward and backward in time, modifying the results of measurements both in past and future. Very notion of the concept of cause and effect, as ordered by time, disappears.

## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

## References


[1] Soiguine, A. (2015) Geometric Phase in Geometric Algebra Qubit Formalism. LAMBERT Academic Publishing, Saarbrucken.

[2] Soiguine, A. (2015) Geometric Algebra, Qubits, Geometric Evolution, and All That. http://arxiv.org/abs/1502.02169

[3] Soiguine, A. (2014) What Quantum "State" Really Is?


[6]It can be verified, though tediously to calculate, that it remains true for any arbitrary Clifford translation plane.

http://arxiv.org/abs/1406.3751

[4] Soiguine, A. (2018) The Geometric Algebra Lift of Qubits via Basic Solutions of Maxwell Equation. https://arxiv.org/abs/1805.11953

[5] Soiguine, A.M. (1996) Complex Conjugation—Relative to What? Clifford Algebras with Numeric and Symbolic Computations, Boston, Birkhauser, 284-294.