Parallelization with g-qubits

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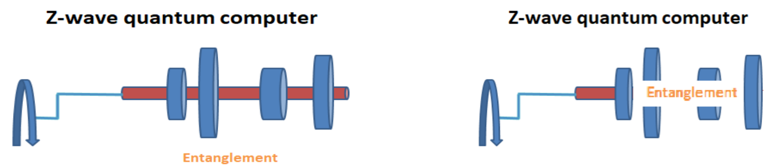
Abstract: Recently appeared ideas about back in time entanglement [1] and indefinite event casual order [2] can be effectively implemented with the help of physically feasible mechanism of g-qubits [3]. The mechanism uses the g-qubit states as solution of Maxwell equations in terms of geometric algebra, along with clear definition of a complex planes as bivectors in three dimensions. This formalism replaces conventional quantum mechanics states, namely objects constructed in the complex vector Hilbert space framework, by the framework of elements of even subalgebra of geometrical algebra G_3^+ which play the role of operators acting on observables [4].

1. Introduction

Quantum computers and quantum cryptography are potentially the first commercial applications of quantum physics. Unfortunately, quantum mechanics in its existing formulation is a no-work-around obstacle to bring these potential applications into reality. The theory needs to be reformulated in adequate logical and mathematical formalism. A lot of continuing confusion comes from the lack of precision in using terms like “state”, “observable”, “measurement of observable in a state”, etc. This terminology creates ambiguity because the meaning of the words differs between prevailing quantum mechanics and what is logically and naturally assumed by the human mind in scientific researches and generally used in areas of physics other than conventional quantum mechanics. We see no successful, reliable results yet in creating quantum computers due to working with bad, inappropriate tools.



Absolutely necessary modifications are clear redefinitions of the “state”, “observable”, “measurement of observable” and “result (value) of measurement of observable” notions [4], [5].

The new definitions follow general paradigm:

- Measurement of observable $O(\mu)$ in state[1] $S(\lambda)$ is a map:

$$\big(S(\lambda), O(\mu)\big) \longrightarrow O(\nu),$$

where $O(\mu)$ is an element of the set of observables. $S(\lambda)$ is element of another set, set of states.

[1] Correctly would be to say “by a state”. State is operator acting on observable.

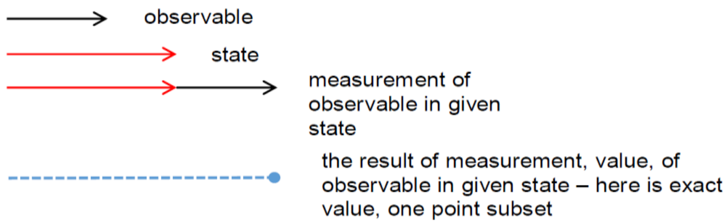
- The result (value) of a measurement of observable $O(\mu)$ by the state $S(\lambda)$ is a map sequence

$$\big(S(\lambda), O(\mu)\big) \longrightarrow\ O(\nu) \longrightarrow V(B),$$

where V is a set of (Boolean) algebra subsets identifying possible results of measurements.

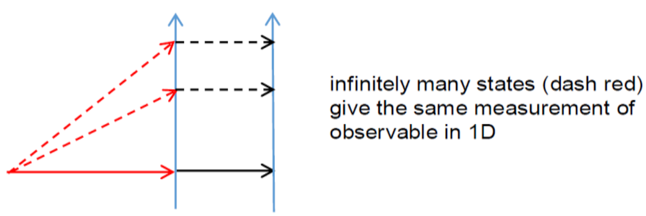
The importance of the above definitions becomes obvious even from trivial examples.

Let's take a point moving along straight line:

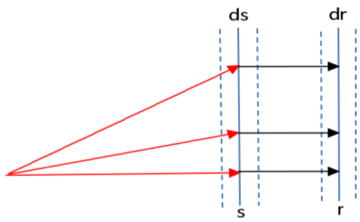


In this classical mechanics example, it does not matter do we consider evolution of "state" or of "measurement of observable by the state" or of "the result of measurement".

The situation radically changes if the process entities become belonging to a plane, though we can only see them in one dimensional projection:



Further, if the evolution is not fully deterministic due to probability distribution of states then probabilities of the results of measurements are obviously calculatable as relative measures of subsets of the level sets (fibers) of states giving the same result:



Probability to get result of measurement in interval **dr** around **r** (senseless "find system in state **r**" as in conventional quantum mechanics) is the integral of probability density of states over the strip **ds**.

The option to expand, to lift the space where physical processes are considered, may have critical consequence to the theory. A kind of expanding is the core of the suggested formulation aimed at the theory that is deeper than conventional quantum mechanics.

Let's take more sophisticated example of expanding the scene of physical situation and get fibration of quantum mechanical qubits to g-qubits, fibers (level sets), elements of even subalgebra of geometrical algebra G_3^+.

Any C^2 qubit $\begin{pmatrix} x_1+iy_1 \\ x_2+iy_2 \end{pmatrix}$ has lift in G_3^+ [5]:

$$x_1 + y_1 B_1 + y_2 B_2 + x_2 B_3 = x_1 + y_1 B_1 + y_2 B_1 B_3 + x_2 B_3 = x_1 + y_1 B_1 + (x_2 + y_2 B_1) B_3$$

where $\{B_1, B_2, B_3\}$ is an arbitrary triple of unit value bivectors in three dimensions satisfying, with not critical assumption of right-hand screw orientation $B_1 B_2 B_3 = 1$, the multiplication rules:

$$B_1 B_2 = -\, B_3,\ B_1 B_3 = B_2,\ B_2 B_3 = -\, B_1$$

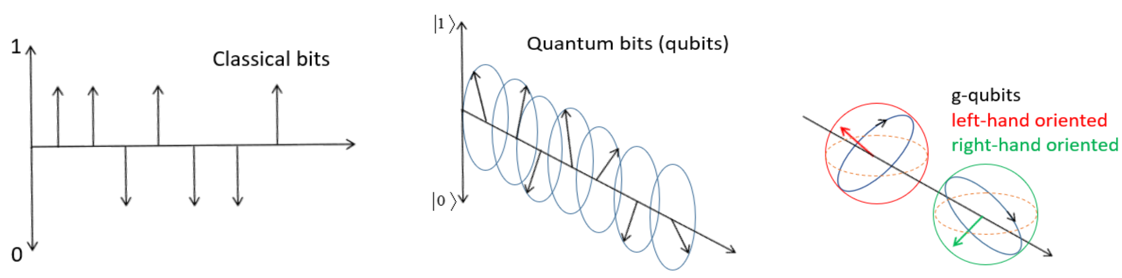
The lift uses the $\{B_1, B_2, B_3\}$ reference frame which can be arbitrary rotated in three dimensions. In that sense we have principal fiber bundle $G_3^+ \to C^2$ with the standard fiber as group of rotations which is also effectively identified by elements of G_3^+.

The lift $x_1 + y_1 B_1 + (x_2 + y_2 B_1) B_3$ is the geometric algebra sum of two items, $x_1 + y_1 B_1$ and $(x_2 + y_2 B_1) B_3$, the first one is lift of the quantum mechanical state $|0\rangle$, in usual Dirac notations, and the second one – lift of $|1\rangle$. The action of a G_3^+ state on observable, also element of algebra G_3, is not as simple as vector translations in the above example. The state action is

$$\big(S(\lambda), O(\mu)\big) \longrightarrow\ O(\nu) \overset{def}{\longleftrightarrow} O(\nu) = S^{-1}(\lambda)\, O(\mu) S(\lambda)$$

The action is not distributive since the results of measurement is not linear combination of measurements by $x_1 + y_1 B_1$ and $(x_2 + y_2 B_1) B_3$. The first one makes rotation in plane B_1 while the second one flips the result after rotation in B_1 over that plane. Nevertheless, any state $x_1 + y_1 B_1 + (x_2 + y_2 B_1) B_3$ can be reduced to either lift, fiber, of $|0\rangle$ or $|1\rangle$ in properly chosen bivector basis [5].

The states as g-qubits store much more information than classical bits or conventional quantum mechanical qubits:



For example, in the quantum cryptography the g-qubit communication scheme can use sequences of g-qubits bearing encoded information. The sequences can be effectively created by instant Clifford translations of original states using some unit value bivector(s) and angle(s). The planes of bivectors can be fixed or varying, as well as the angles. These parameters of Clifford translations comprise the “secret key”. On the other end of the communication channel the receiver needs this key to decode the sequence. In the simplest protocol the key is just the fixed plane definition and the value of angle. In the current theory the same key works both in encoding and decoding.

2. Action of an arbitrary plane Clifford translations on states which are the Maxwell equation solutions

It was shown in [3] that g-qubits implemented via the two basic solutions of Maxwell equations in geometric algebra terms may have the form:

$$\alpha + \beta B_1 + \alpha B_2 + \beta B_3 = \alpha + \beta B_3 + (\alpha + \beta B_3) B_2 \tag{2.1}$$

The (2.1) g-qubit, when scaled to unit value element of G_3^+,

$$\frac{\alpha}{\sqrt{2(\alpha^2+\beta^2)}} + \frac{\beta}{\sqrt{2(\alpha^2+\beta^2)}} B_1 + \frac{\alpha}{\sqrt{2(\alpha^2+\beta^2)}} B_2 + \frac{\beta}{\sqrt{2(\alpha^2+\beta^2)}} B_3$$

can be written in standard exponential form:

$$\cos\varphi + \sin\varphi\, I_B = e^{I_B \varphi} \tag{2.2}$$

In the considered case the (2.1) – (2.2) g-qubits are linear combinations of the two equally weighted basic solutions of the Maxwell equation F_+ and F_-, $\lambda F_+ + \mu F_-$ with $\lambda = \mu = 1$. In this case the general linear combination, see [6], [7], reads:

$$\lambda F_+ + \mu F_-|_{\lambda=\mu=1} = 2\cos\omega[(I_3 I_S)\cdot\vec{r}]\left(\frac{1}{\sqrt{2}}\cos\omega t + I_S\frac{1}{\sqrt{2}}\sin\omega t + I_{B_0}\frac{1}{\sqrt{2}}\cos\omega t + I_{E_0}\frac{1}{\sqrt{2}}\sin\omega t\right) =$$
$$2\cos\omega[(I_3 I_S)\cdot\vec{r}]\left(\cos\varphi + \sin\varphi\left(I_S\frac{\sin\omega t}{\sqrt{1+(\sin\omega t)^2}} + I_{B_0}\frac{\cos\omega t}{\sqrt{1+(\sin\omega t)^2}} + I_{E_0}\frac{\sin\omega t}{\sqrt{1+(\sin\omega t)^2}}\right)\right) \tag{2.3}$$

where $\cos\varphi = \frac{1}{\sqrt{2}}\cos\omega t$ and $\sin\varphi = \frac{1}{\sqrt{2}}\sqrt{1+(\sin\omega t)^2}$. The state (2.3) is of the form (2.2) because $\alpha = \beta_2$ and $\beta_1 = \beta_3$.

I will call that kind of g-qubits ***spreons*** due to their spreading over the whole three-dimensional space for all values of time, along with the results of measurement of any observable.

In the following I use the triple of unit value orthonormal bivectors $\{I_S, I_{B_0}, I_{E_0}\}$ where I_S is bivector spanned by initial vector of magnetic field and initial vector of electric field.

Bivector I_{B_0} is dual to initial vector of magnetic field, $B_0 = I_3\vec{H}_0$[2], and I_{E_0} is dual to initial vector of electric field. The triple will play the role of the bivector basis $\{B_1, B_2, B_3\}$ which becomes fixed in that way.

Good to remember that the two basic solutions F_+ and F_- differ only by the sign of $I_3 I_S$ [6], which is caused by orientation of I_S that in its turn defines if the triple $\{\hat{E}, \hat{H}, \pm I_3 I_S\}$[3] is right-hand screw or left-hand screw oriented [7].

Action of Clifford translation by γ in arbitrary plane B_C on the state (2.2) is, by definition, $e^{I_B\varphi} \to e^{I_{B_C}\gamma} e^{I_B\varphi}$.

Remark 2.1:

If Clifford translation of a state $e^{I_S(t)\varphi(t)}$ is associated with a Hamiltonian, that's the translation is $e^{-I_3\frac{H(t_0)}{|H(t_0)|}|H(t_0)|\Delta t} e^{I_S(t_0)\varphi(t_0)}$, where Hamiltonian $H(t)$ is lift of some Hermitian matrix $\begin{pmatrix} a+b & c-id \\ c+id & a-b \end{pmatrix}$ [6], [7], [8], and $I_3 \frac{H(t_0)}{|H(t_0)|} \equiv I_H(t_0)$ is generalization of imaginary unit in the current theory, then:

$$e^{I_S(t_0+\Delta t)\varphi(t_0+\Delta t)} = e^{-I_H(t_0)|H(t_0)|\Delta t} e^{I_S(t_0)\varphi(t_0)}$$

and $$\lim_{\Delta t \to 0} \frac{\Delta e^{I_S(t_0)\varphi(t_0)}}{\Delta t} = \lim_{\Delta t \to 0} \frac{e^{I_S(t_0+\Delta t)\varphi(t_0+\Delta t)} - e^{I_S(t_0)\varphi(t_0)}}{\Delta t} =$$
$$\lim_{\Delta t \to 0} \frac{(1 - I_H(t_0)|H(t_0)|\Delta t) e^{I_S(t_0)\varphi(t_0)} - e^{I_S(t_0)\varphi(t_0)}}{\Delta t} = -I_H(t_0)|H(t_0)| e^{I_S(t_0)\varphi(t_0)}$$

that immediately gives the Schrodinger equation for the state $e^{I_S(t)\varphi(t)}$. That means that Schrodinger equation governs evolution of operators, states, which act on observables.

End of the Remark 2.1.

Take the spreon (2.3):

$$F_+ + F_- = 2\cos\omega[(I_3 I_S)\cdot\vec{r}]\left(\frac{1}{\sqrt{2}}\cos\omega t + I_S\frac{1}{\sqrt{2}}\sin\omega t + I_{B_0}\frac{1}{\sqrt{2}}\cos\omega t + I_{E_0}\frac{1}{\sqrt{2}}\sin\omega t\right)$$

Its Clifford translation by $e^{I_{B_C}\gamma} = \cos\gamma + \sin\gamma(\gamma_1 I_S + \gamma_2 I_{B_0} + \gamma_3 I_{E_0})$ gives:

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[2] If we have a unit value bivector B then B and I_B can be used equivalently.

[3] For any vector the notation is used: $\hat{a} = \vec{a}/|\vec{a}|$

$$2\cos\omega[(I_3I_S)\cdot\vec{r}]\Big[\frac{1}{\sqrt{2}}\big(\cos\gamma\cos\omega t-\gamma_1\sin\gamma\sin\omega t-\gamma_2\sin\gamma\cos\omega t-\gamma_3\sin\gamma\sin\omega t\big)+$$
$$\frac{1}{\sqrt{2}}\big(\cos\gamma\sin\omega t+\gamma_1\sin\gamma\cos\omega t-\gamma_2\sin\gamma\sin\omega t+\gamma_3\sin\gamma\cos\omega t\big)I_S+$$
$$\frac{1}{\sqrt{2}}\big(\cos\gamma\cos\omega t+\gamma_1\sin\gamma\sin\omega t+\gamma_2\sin\gamma\cos\omega t-\gamma_3\sin\gamma\sin\omega t\big)I_{B_0}+$$
$$\frac{1}{\sqrt{2}}\big(\cos\gamma\sin\omega t-\gamma_1\sin\gamma\cos\omega t+\gamma_2\sin\gamma\sin\omega t+\gamma_3\sin\gamma\cos\omega t\big)I_{E_0}\Big] \qquad (2.4)$$

that is defined for all values of t and $\vec{r}$, in other words spreads through the whole three-dimensions at any instant of time.

The Hopf fibration, measurement of any observable $C_0+C_1B_1+C_2B_2+C_3B_3$ in the current formalism (see [8], Sec.5.1):

$$\begin{aligned}C_0+C_1B_1+C_2B_2+C_3B_3 &\xrightarrow{\alpha+\beta_1B_1+\beta_2B_2+\beta_3B_3} C_0\\ &+\big(C_1[(\alpha^2+\beta_1^2)-(\beta_2^2+\beta_3^2)]+2C_2(\beta_1\beta_2-\alpha\beta_3)+2C_3(\alpha\beta_2+\beta_1\beta_3)\big)B_1\\ &+\big(2C_1(\alpha\beta_3+\beta_1\beta_2)+C_2[(\alpha^2+\beta_2^2)-(\beta_1^2+\beta_3^2)]+2C_3(\beta_2\beta_3-\alpha\beta_1)\big)B_2\\ &+(2C_1(\beta_1\beta_3-\alpha\beta_2)+2C_2(\alpha\beta_1+\beta_2\beta_3)+C_3[(\alpha^2+\beta_3^2)-(\beta_1^2+\beta_2^2)])B_3\end{aligned}$$

with:

$B_1=I_S$, $B_2=I_{B_0}$, $B_3=I_{E_0}$,

$$\alpha=2\cos\omega[(I_3I_S)\cdot\vec{r}]\frac{1}{\sqrt{2}}(\cos\gamma\cos\omega t-\gamma_1\sin\gamma\sin\omega t-\gamma_2\sin\gamma\cos\omega t-\gamma_3\sin\gamma\sin\omega t)$$

$$\beta_1=2\cos\omega[(I_3I_S)\cdot\vec{r}]\frac{1}{\sqrt{2}}(\cos\gamma\sin\omega t+\gamma_1\sin\gamma\cos\omega t-\gamma_2\sin\gamma\sin\omega t+\gamma_3\sin\gamma\cos\omega t)$$

$$\beta_2=2\cos\omega[(I_3I_S)\cdot\vec{r}]\frac{1}{\sqrt{2}}(\cos\gamma\cos\omega t+\gamma_1\sin\gamma\sin\omega t+\gamma_2\sin\gamma\cos\omega t-\gamma_3\sin\gamma\sin\omega t)$$

$$\beta_3=2\cos\omega[(I_3I_S)\cdot\vec{r}]\frac{1}{\sqrt{2}}(\cos\gamma\sin\omega t-\gamma_1\sin\gamma\cos\omega t+\gamma_2\sin\gamma\sin\omega t+\gamma_3\sin\gamma\cos\omega t)$$

gives a G_3^+ element $O\big(C_0,C_1,C_2,C_3,I_S,I_{B_0},I_{E_0},\gamma,\gamma_1,\gamma_2,\gamma_3,\omega,t,\vec{r}\big)$ with scalar component C_0 and bivector components being up to the second order polynomials of $\sin\gamma$, $\cos\gamma$, $\sin\omega t$ and $\cos\omega t$ that spread through the whole three-dimensional space at any time, all components multiplied by $4(cos\ \omega\ [(I_3I_S)\cdot\vec{r}])^2$.

3. Parallelism of computational algorithm

The instant of time when the Clifford translation is applied makes no difference for the resulting state (2.4) because it simultaneously is redefined for all values of t. We have changing of the state also backward in time. That is obvious demonstration that the suggested theory allows indefinite event casual order. In that way, as assumed by the physicists working on the causal asymmetry problems [9], the very notion of the concept of cause and effect disappears, thus we might not perceive time.

Further, we get superiority of the suggested theory over classical computations because all measured observable values get available all together, not through looking one by one. Simultaneous availability has always been the proof of calculational supremacy of hypothetical qubit entangled quantum computers. The current approach transcends those computational schemes also since the latter have tough problems of creating large sets of qubits. In the current scheme any number of test observables can be placed into continuum of the $(t, \vec{r})$ dependent values of the spreon state, thus fetching out any amount of values $O\big(C_0, C_1, C_2, C_3, I_S, I_{B_0}, I_{E_0}, \gamma, \gamma_1, \gamma_2, \gamma_3, \omega, t, \vec{r}\big)$, spread over three-dimensions and at all instants of time not generally following cause/effect ordering.

As I mentioned once, quantum computer built in the current theory approach is a kind of analogue computer, aimed, first of all, at modeling three-dimensional vector fields.

Since general result of measurement when Clifford translation takes place in an arbitrary plane is pretty complicated I am taking first the special case $\gamma_1 = 1$ and $\gamma_2 = \gamma_3 = 0$ (Clifford translation acts in plane I_S). The result is:

$$\begin{aligned} &O\big(C_0, C_1, C_2, C_3, I_S, I_{B_0}, I_{E_0}, \gamma, \gamma_1, \gamma_2, \gamma_3, \omega, t, \vec{r}\big)_{\gamma_1=1,\gamma_2=\gamma_3=0} \\ &\quad = 4(cos\,\omega\,[(I_3 I_S)\cdot\vec{r}])^2\big[C_0 + (C_2\,sin\,2\gamma + C_3\,cos\,2\gamma)I_S \\ &\quad + \big(C_1\,sin\,2\omega t + sin\,2\gamma\,cos\,2\omega t\,(C_2 + C_3)\big)I_{B_0} \\ &\quad + (-C_1\,cos\,2\omega t + sin\,2\gamma\,sin\,2\omega t\,(C_2 - C_3))I_{E_0}\big] \end{aligned}$$

Interesting thing is that the component of measurement lying in plane I_S is only defined by the applied Clifford translation parameters and does not change with time [4]. It only depends on the $\vec{r}$ value and Clifford translation parameter γ.

In two other cases $\gamma_2 = 1$, $\gamma_1 = \gamma_3 = 0$ and $\gamma_3 = 1$, $\gamma_1 = \gamma_2 = 0$ we get correspondingly:

$$\begin{aligned} &O\big(C_0, C_1, C_2, C_3, I_S, I_{B_0}, I_{E_0}, \gamma, \gamma_1, \gamma_2, \gamma_3, \omega, t, \vec{r}\big)_{\gamma_2=1,\gamma_1=\gamma_3=0} \\ &\quad = 4(cos\,\omega\,[(I_3 I_S)\cdot\vec{r}])^2\big[C_0 + (-C_1\,sin\,2\gamma + C_3\,cos\,2\gamma)I_S \\ &\quad + \big(C_2\,cos\,2\omega t + sin\,2\omega t\,(C_1\,cos\,2\gamma + C_3 sin\,2\gamma)\big)I_{B_0} \\ &\quad + \big(C_2\,sin\,2\omega t - cos\,2\omega t\,(C_1\,cos\,2\gamma + C_3 sin\,2\gamma)\big)I_{E_0}\big] \end{aligned}$$

and

$$\begin{aligned} &O\big(C_0, C_1, C_2, C_3, I_S, I_{B_0}, I_{E_0}, \gamma, \gamma_1, \gamma_2, \gamma_3, \omega, t, \vec{r}\big)_{\gamma_3=1,\gamma_1=\gamma_2=0} \\ &\quad = 4(cos\,\omega\,[(I_3 I_S)\cdot\vec{r}])^2\big[C_0 + C_3 I_S \\ &\quad + \big(C_1(cos\,2\gamma\,sin\,2\omega t + sin\,2\gamma\,cos\,2\omega t) \\ &\quad + C_2(cos\,2\gamma\,cos\,2\omega t - sin\,2\gamma\,sin\,2\omega t)\big)I_{B_0} \\ &\quad + \big(C_1(sin\,2\gamma\,sin\,2\omega t - cos\,2\gamma\,cos\,2\omega t) \\ &\quad + C_2(sin\,2\gamma\,cos\,2\omega t + cos\,2\gamma\,sin\,2\omega t)\big)I_{E_0}\big] \end{aligned}$$

[4] It can be verified, though tediously to calculate, that it remains true for any arbitrary Clifford translation plane.

In any case we get bivector, that is equivalent to a three-dimensional vector at point $\vec{r}$ and arbitrary value of time which is in no way related to the instant of time of the Clifford translation action on the state. The values of the bivector components are defined by the measured observable components, vector $\vec{r}$ of the point where the observable is placed, time of measurement (all of them comprise the input of the computational algorithm), along with Clifford translation parameters.

4. Conclusions

Two seminal ideas – variable and explicitly defined complex plane in three dimensions, and the G_3^+ states[5] as operators acting on observables – allow to put forth comprehensive and much more detailed formalism appropriate for quantum mechanics in general and particularly for quantum computing schemes.

Solution of the Maxwell equation(s) in the G_3 frame particularly gives G_3^+ state, spreon, spreading over the whole three-dimensional space for all values of time, along with the results of measurement of any observable. The state can particularly change instantly backward in time under Clifford translations. Very notion of the concept of cause and effect disappears, thus we might not perceive time. Computations, executed through Clifford translations, return all measured observable values all together. Any number of test observables can be placed into continuum of the $(t, \vec{r})$ dependent values of the spreon state, thus fetching out any amount of values.

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[5] Good to remember that “state” and “wave function” are (at least should be) synonyms in conventional quantum mechanics

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